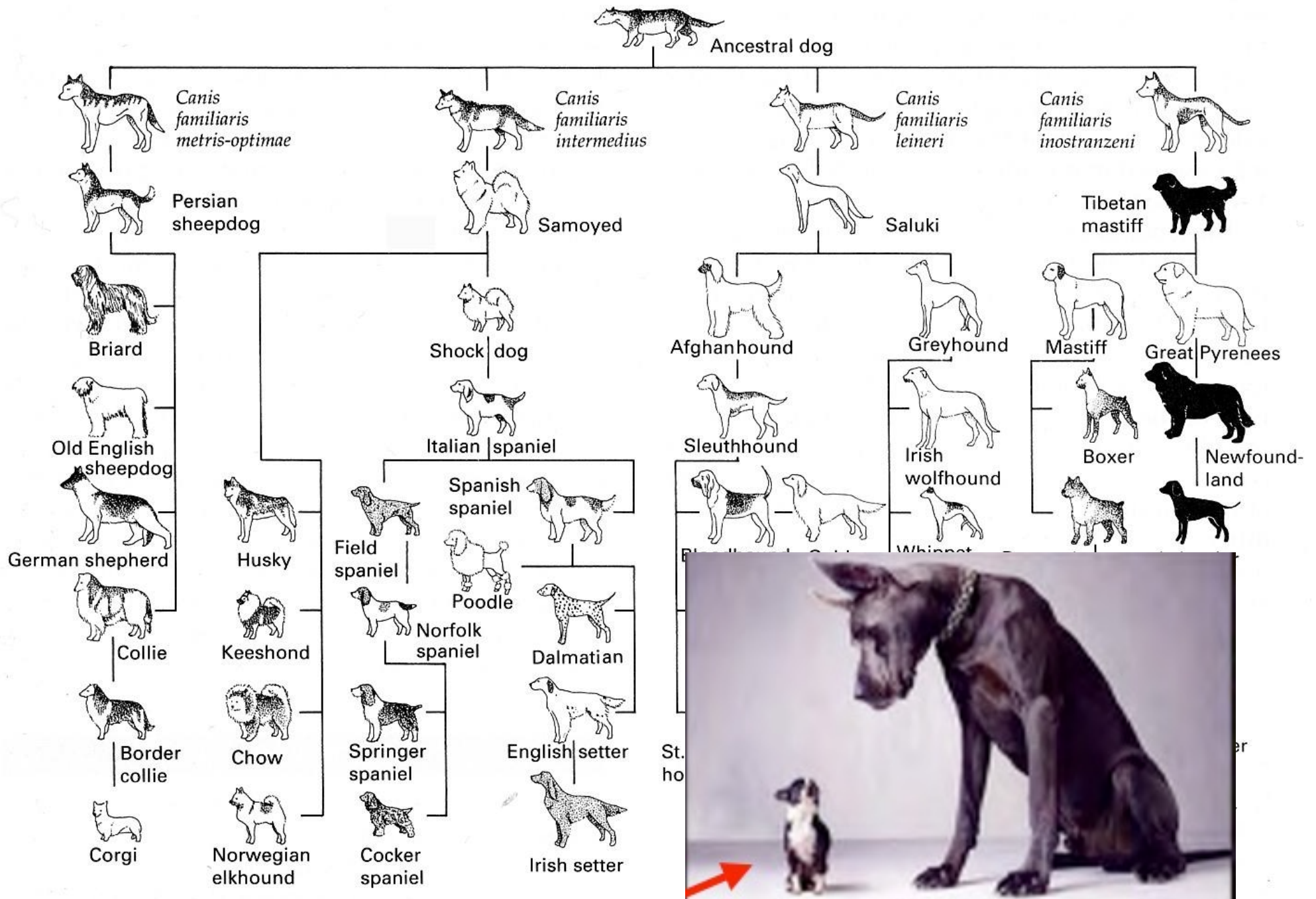


**Lecture 4**  
**Quantitative traits and**  
**Mapping, Mapping, Mapping!!!**

# Quantitative Traits

- ...traits that show a continuous variation in phenotype over a range,
- ...often result from multiple genes and are further termed polygenic traits.





# *Brassica oleracea*

All these foods are the same species



Cauliflower



Cabbage



Brussels sprouts

Flower clusters

Terminal bud

Lateral buds



Mustard

Flowers and stems

Leaves

Stem



Broccoli



Kale



Kohlrabi

# Discontinuous Traits

- Discontinuous Traits,
  - exhibit only a few distinct phenotypes and can be described in a qualitative manner,
- Mendel and others worked with “inbred” lines,

Father: DD BB cc aa EE FF GG

Mother: dd BB cc aa EE FF GG

# A Continuous Trait with a major component. Dog breed size

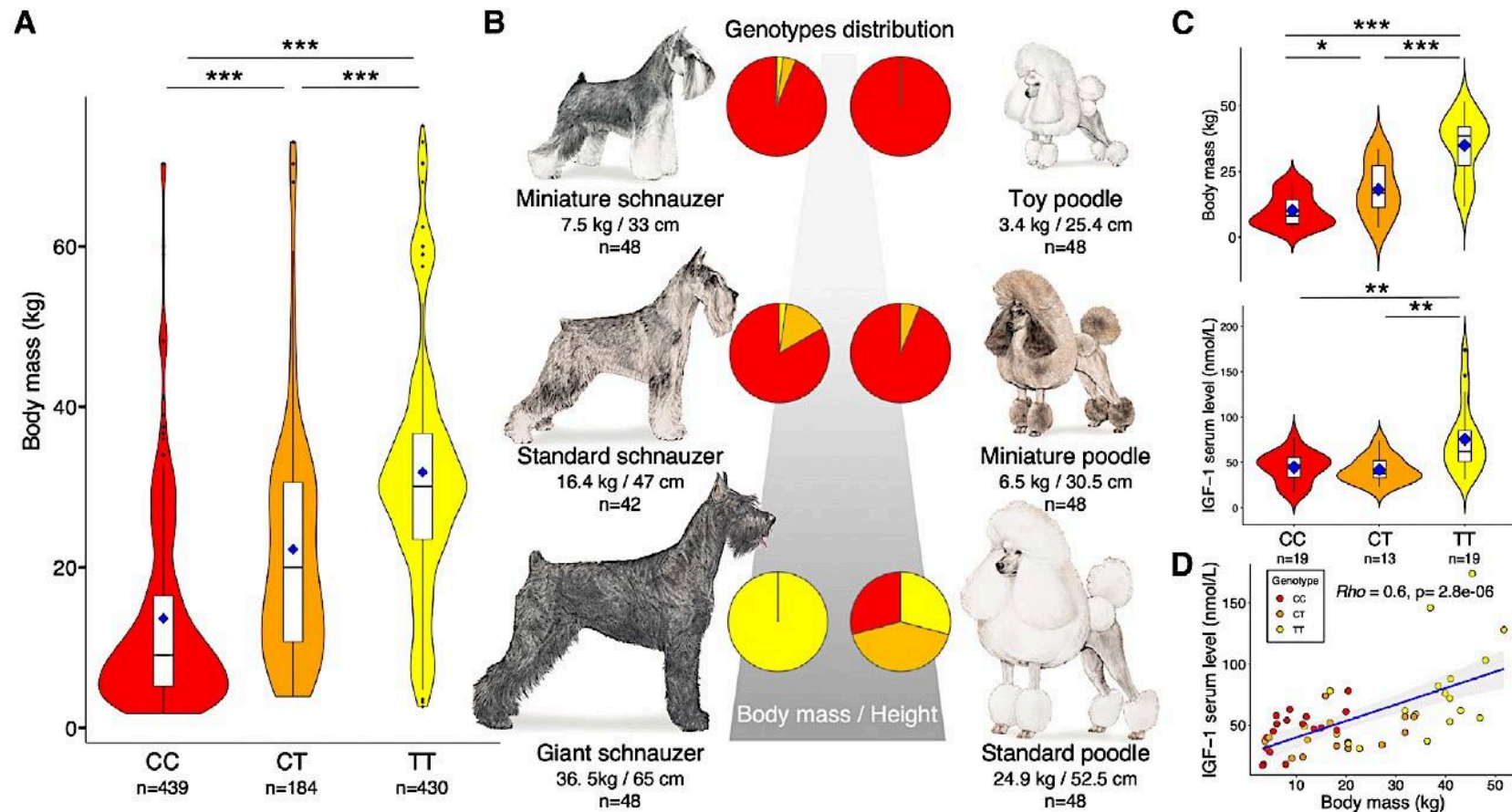
- “Outbred” transmission,

|         |       |    |    |    |       |
|---------|-------|----|----|----|-------|
| Father: | dd BB | CC | AA | EE | FF GG |
| Mother: | DD bb | CC | aa | EE | ff qq |

- Continuous Traits,
  - Display a spectrum of phenotypes, and must be described in quantitative terms,

<https://doi.org/10.1016/j.cub.2021.12.036>





**Figure 1. Insulin-like growth factor 1 (IGF1) in Canidae**

(A) *IGF1-AS* variant genotypes and body mass range collected from 1,162 dogs of 230 breeds. Dots represent outliers. Blue diamonds indicate breed body mass averages, boxplots represent interquartile ranges, and black horizontal bars show median for each (\*\*p < 0.0001, Mann-Whitney-Wilcoxon tests).

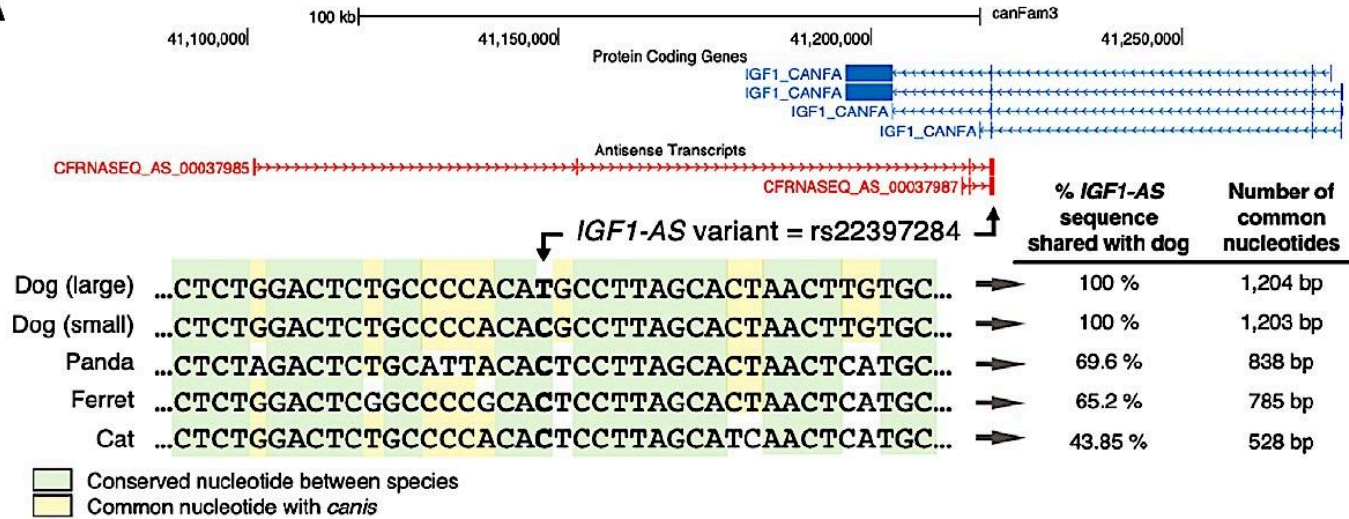
(B) Distribution of *IGF1-AS* alleles in three schnauzer breeds and poodle varieties. Pie chart indicates population proportion based on genotypes. Red, CC; orange, CT; yellow, TT.

(C) Body mass and serum levels of IGF-1 protein (nmol/L) as functions of *IGF1-AS* genotype. IGF-1 serum protein levels were assayed in 51 dogs, including 13 mixed-breed dogs (\*p < 0.01, \*\*p < 0.001, \*\*\*p < 0.0001, Mann-Whitney-Wilcoxon tests).

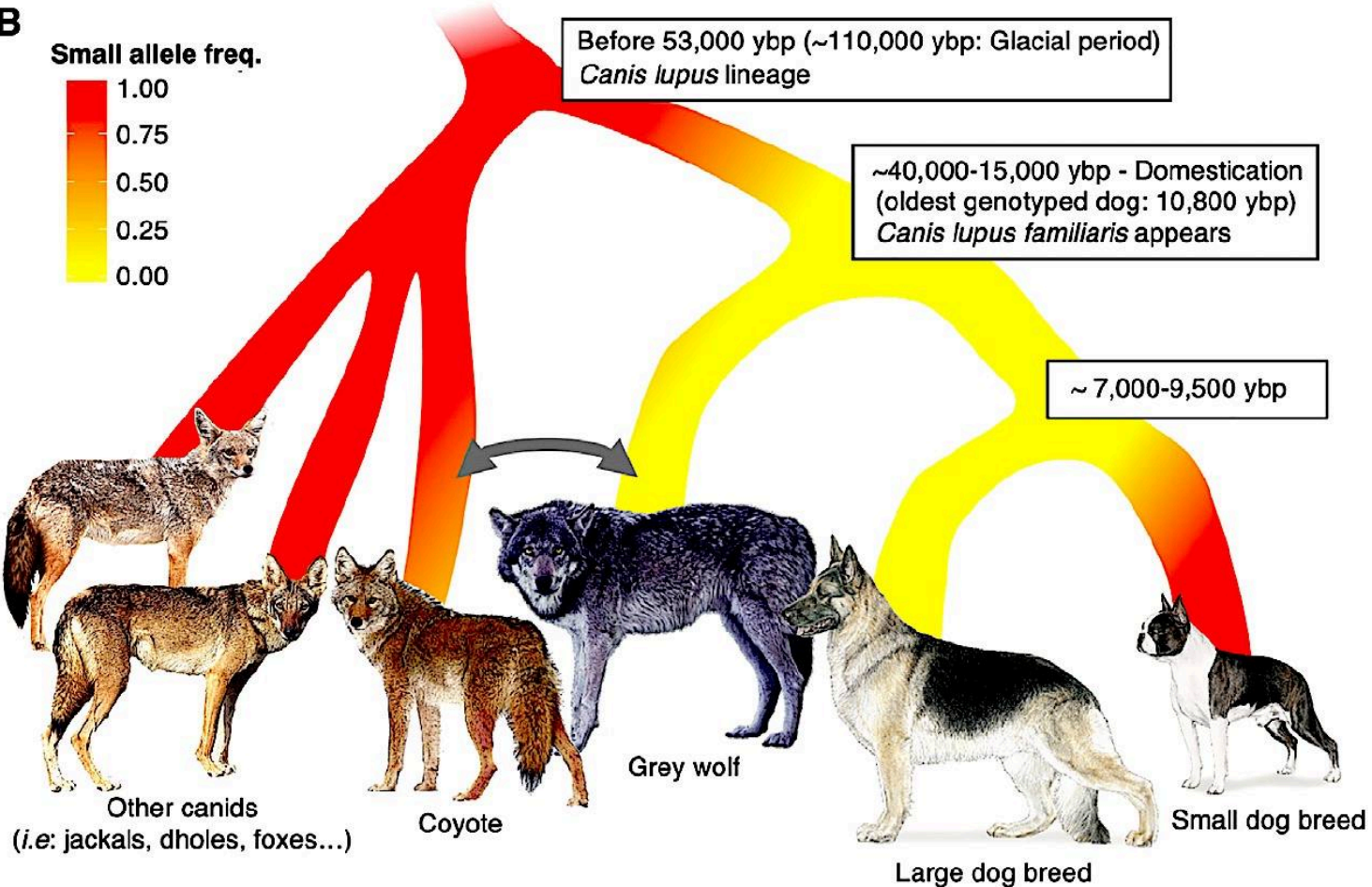
(D) Positive correlation observed between body mass and IGF-1 serum level (*Rho* Spearman test). Blue line shows the regression line; gray area represents confidence interval.

See also [Table S1](#) and [Data S1](#).

**A**

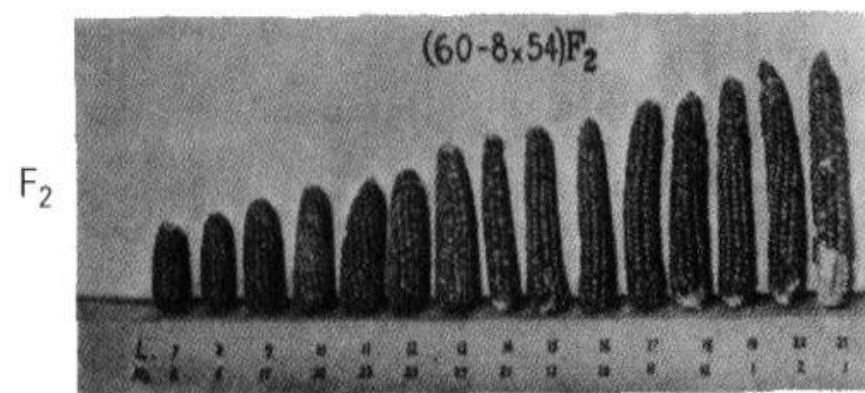
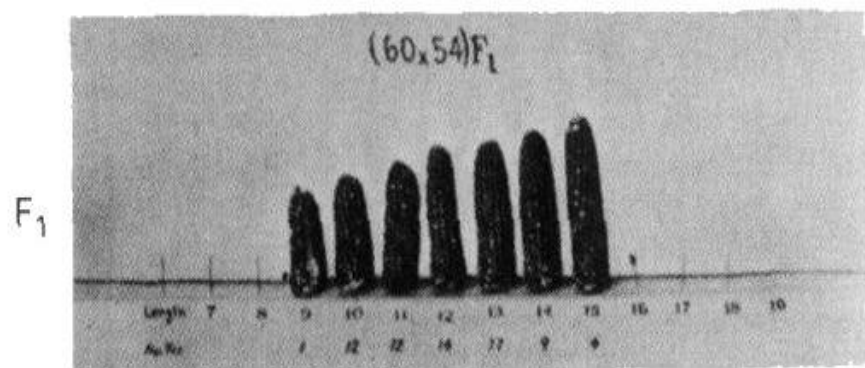
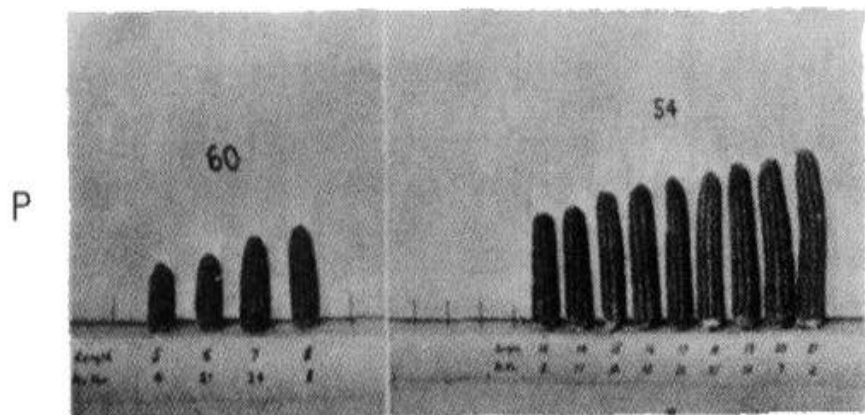


**B**

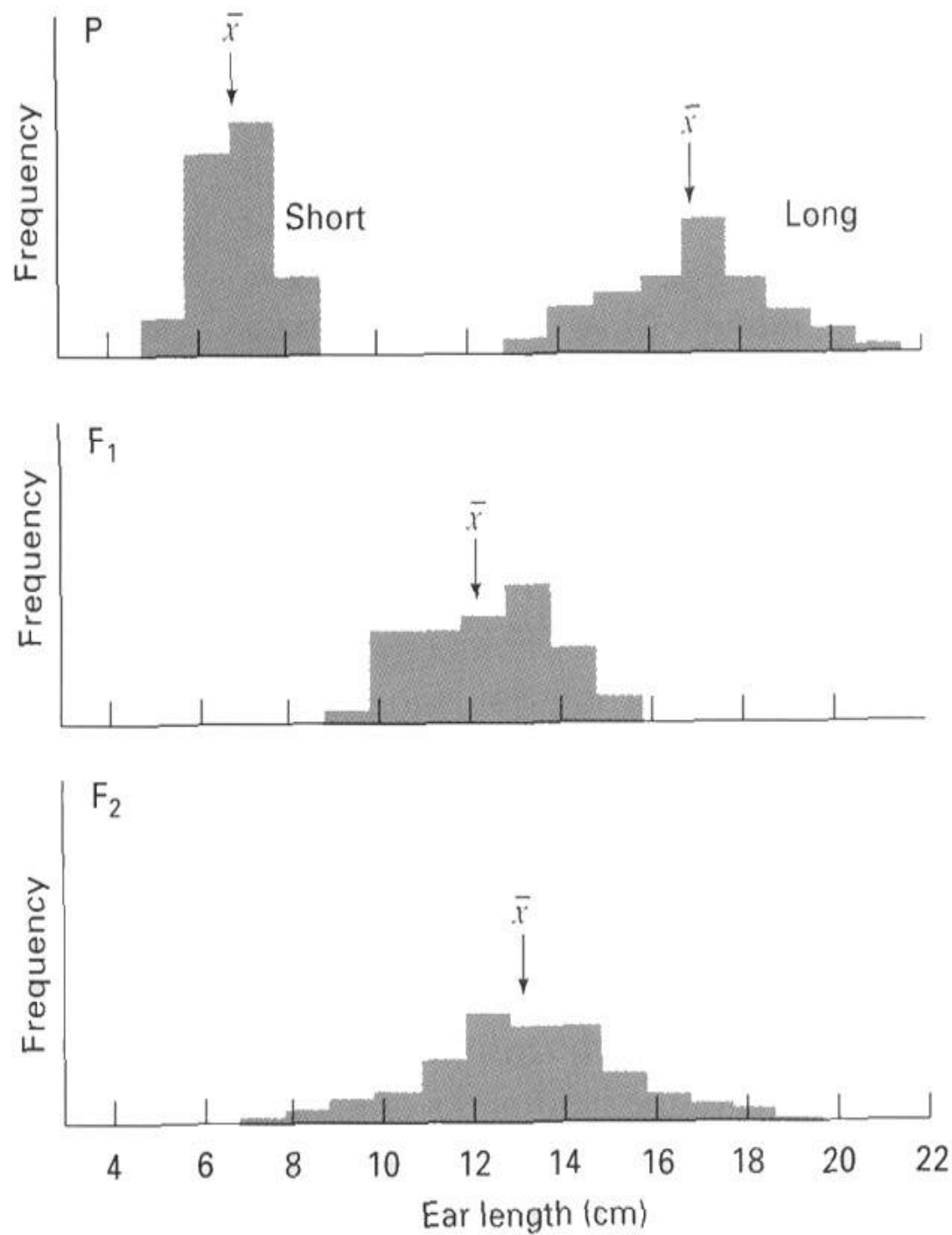


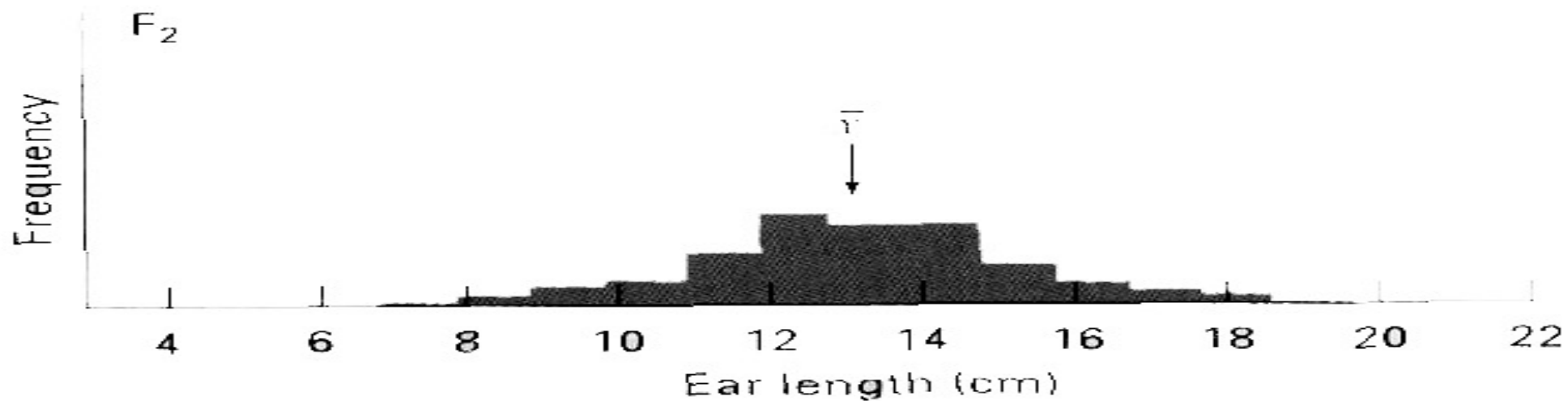


a)

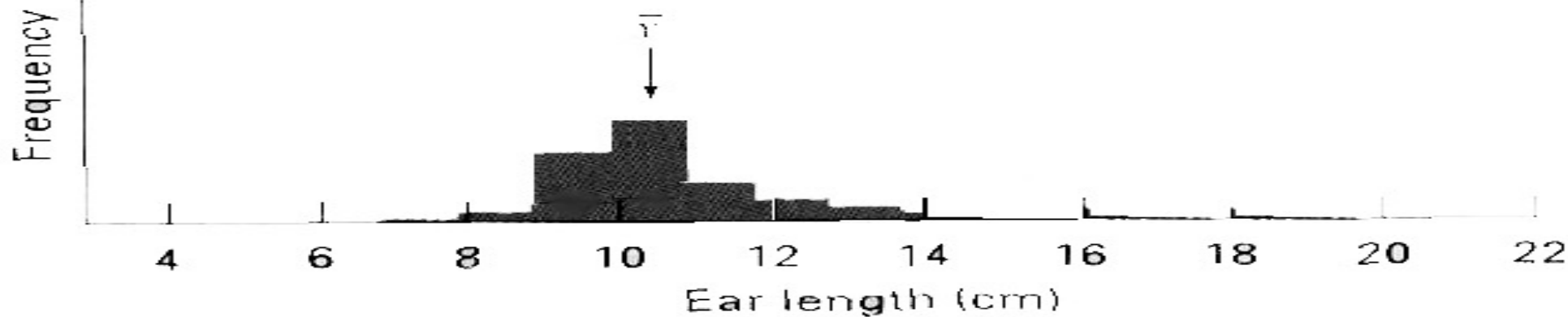


b)

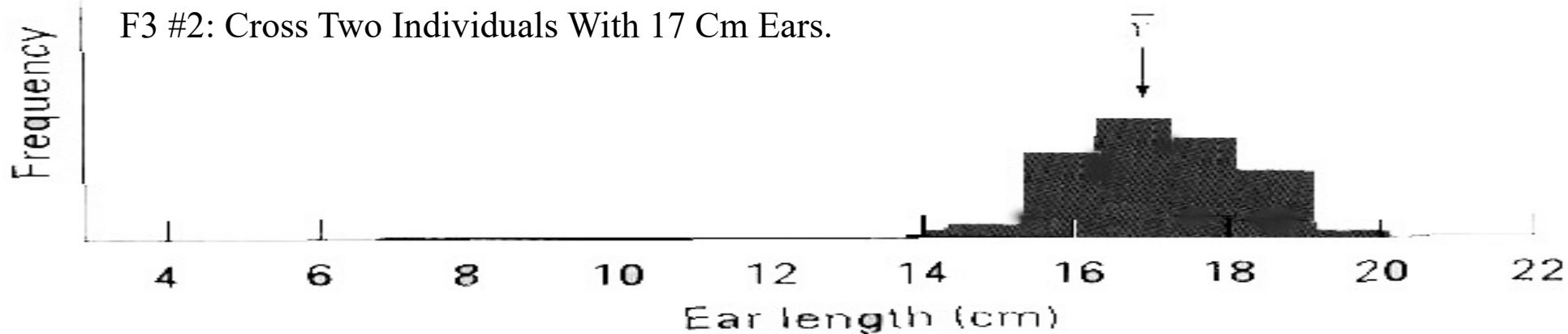




F3 #1: Cross Two Individuals With 10 Cm Ears.



F3 #2: Cross Two Individuals With 17 Cm Ears.

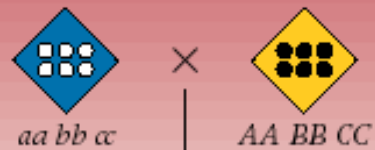


# Polygenic Traits and Mendel

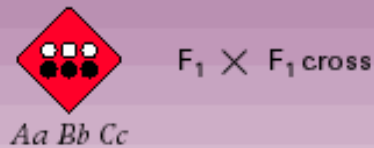
- It is possible to provide a Mendelian explanation for continuous variation by considering numbers of genes contributing to a phenotype,
  - the more genes, the more phenotypic categories,
  - the more categories, the more the variation seems continuous.



Parents:

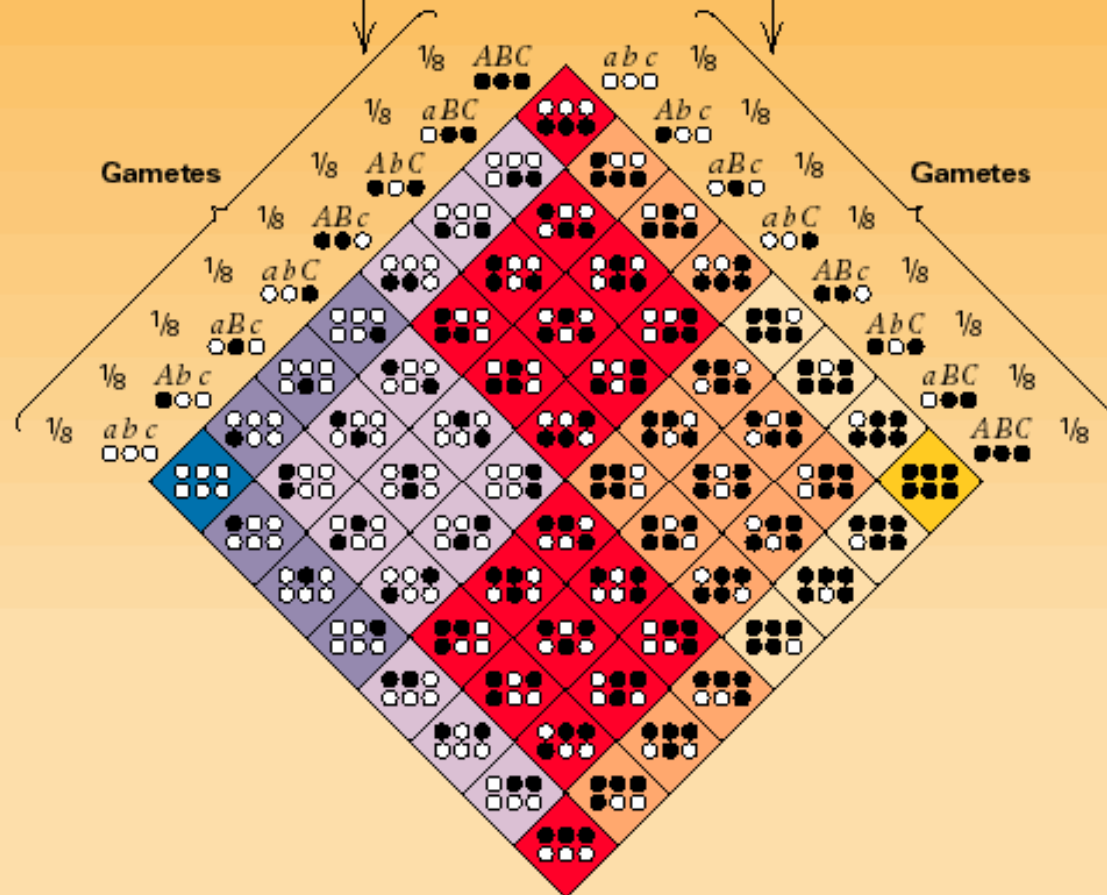


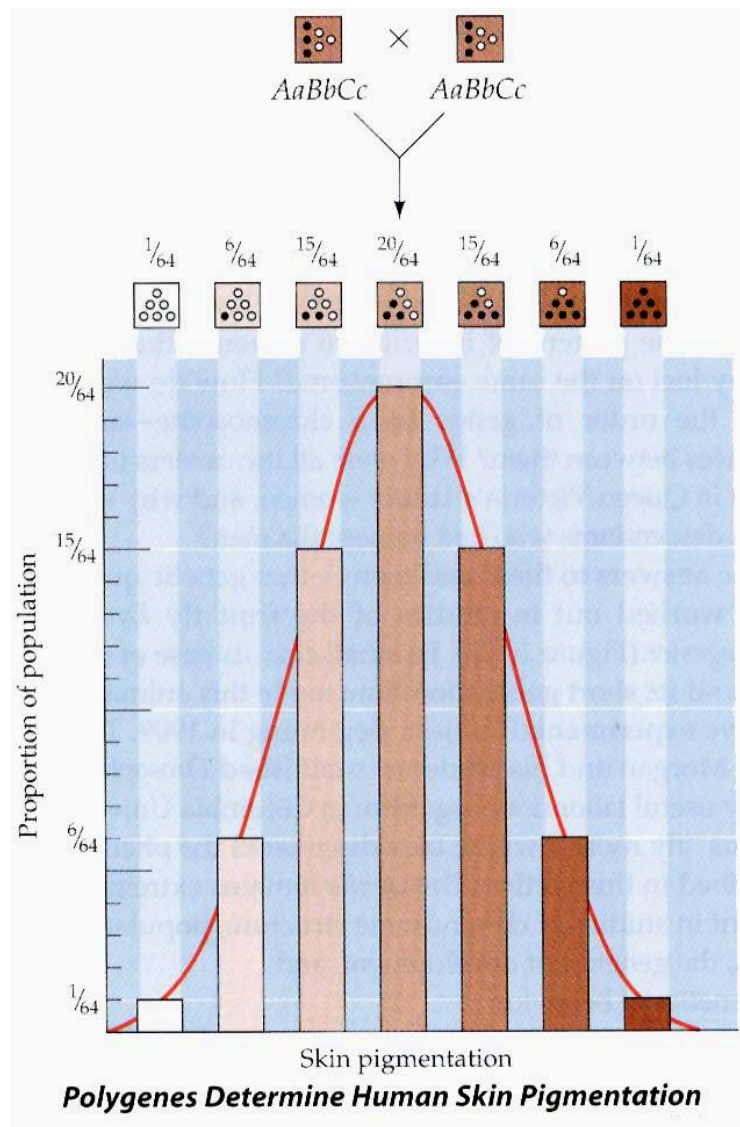
F<sub>1</sub> progeny:



F<sub>2</sub> progeny:

Mendelian segregation and recombination



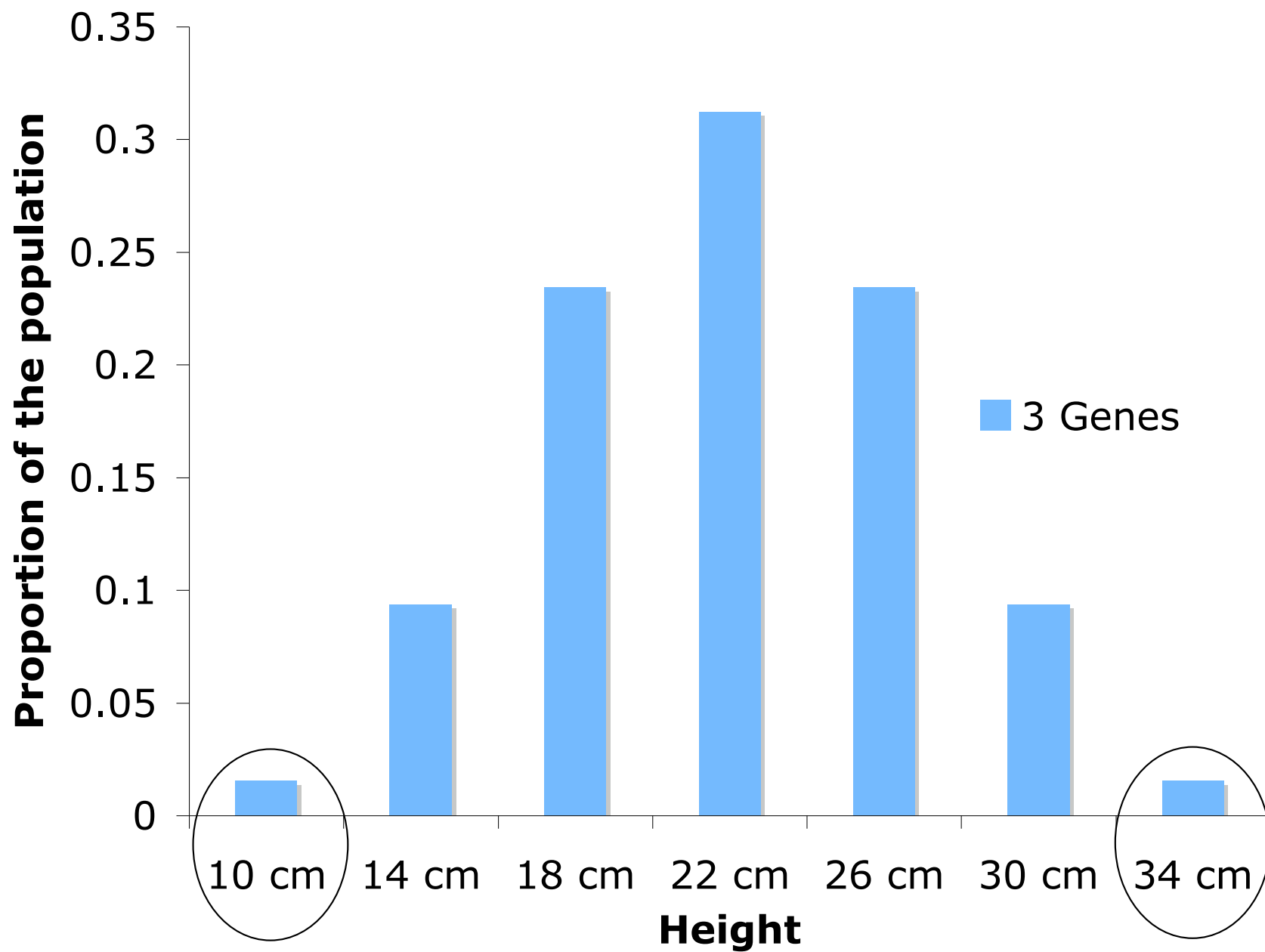


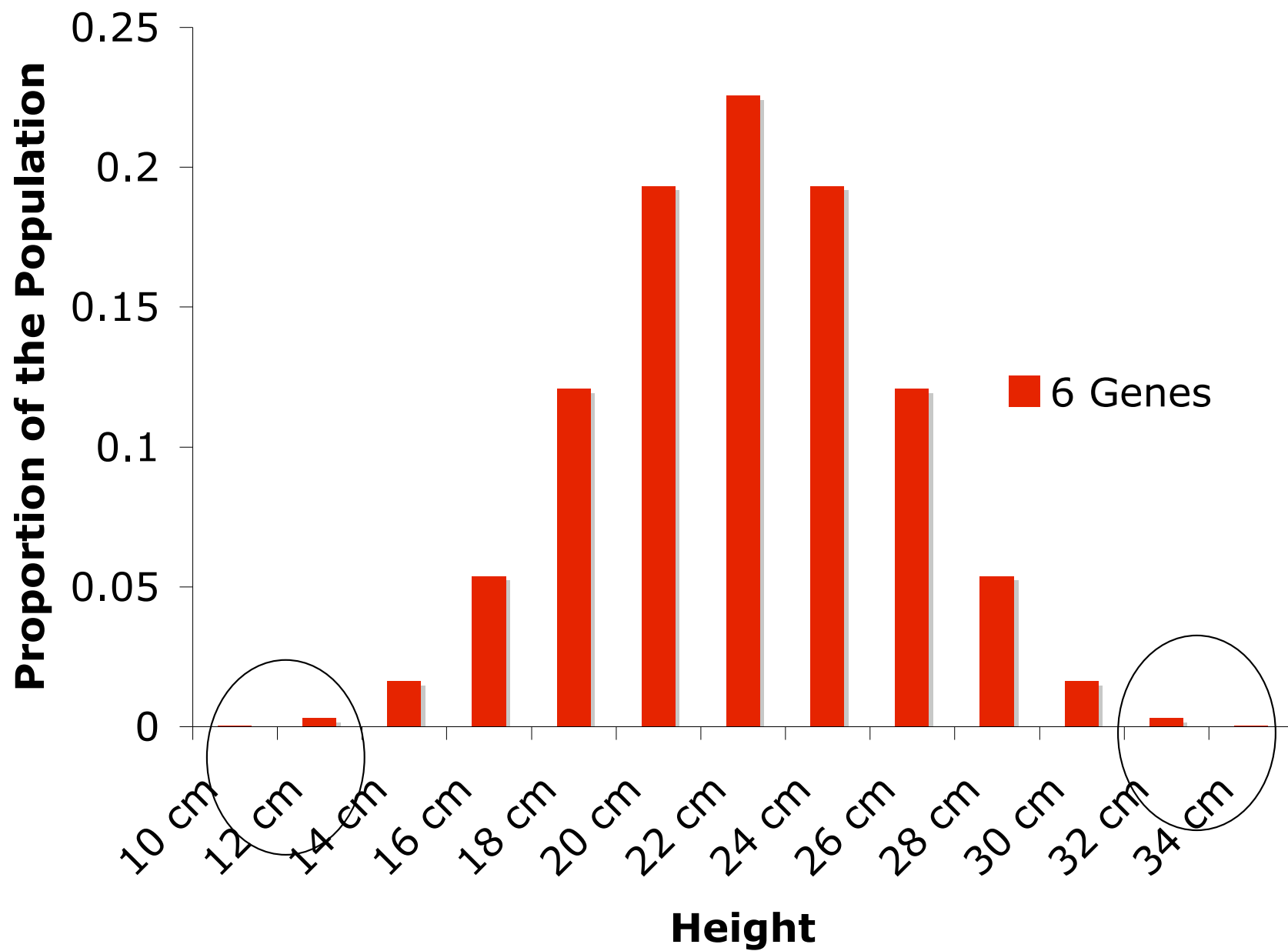
## Melanin Pigmentation in twins

**Twins, One White and One Black, Get Ready to Start Middle School: 'I Notice People Doing Double-Takes'**

INSIDE edition Inside Edition 9 hours ago









# Continuous Variation

- Two or more genes that influence the same phenotype, often in an additive way,
  - **additive allele:** contributes a set amount to the phenotype,
  - **non-additive allele:** does not contribute to the phenotype.
- The affect of each allele on the phenotype is relatively small, and roughly equivalent,
- Substantial variation is observed when multiple genes control a single trait,
- Must be studied in large populations.

# Calculating the Number of Genes

- If you know the frequency of *either* extreme trait, then the number of genes ( $n$ ) can be calculated using the formulae...

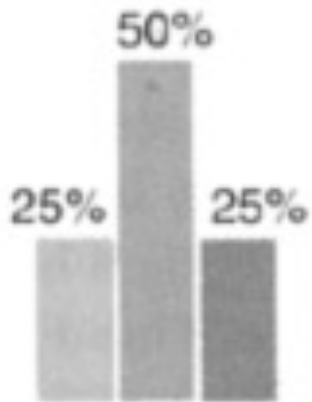
$$\frac{1}{4^n} = \text{ratio of F2 individuals expressing} \\ \text{either extreme phenotype.}$$

## The number of genes: $n = 1$

|       | $A^1$ | $A^0$ |
|-------|-------|-------|
| $A^1$ | 2     | 1     |
| $A^0$ | 1     | 0     |

1/4 show either extreme trait.

$$\frac{1}{4^n} = \frac{1}{4^1}$$



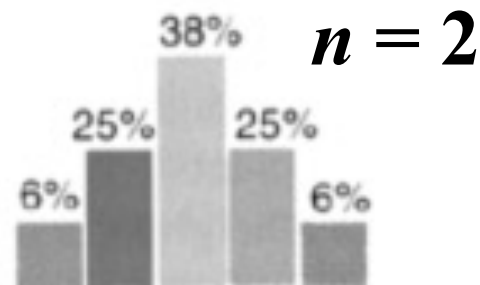
$n = 1$

## The number of genes: $n = 2$

|          | $A^1B^1$ | $A^1B^0$ | $A^0B^1$ | $A^0B^0$ |
|----------|----------|----------|----------|----------|
| $A^1B^1$ | 4        | 3        | 3        | 2        |
| $A^1B^0$ | 3        | 2        | 2        | 1        |
| $A^0B^1$ | 3        | 2        | 2        | 1        |
| $A^0B^0$ | 2        | 1        | 1        | 0        |

1/16 show either Extreme trait.

$$\frac{1}{4^n} = \frac{1}{16} = \frac{1}{4^2}$$



$n = 2$

- Inbred strain #1, mean height = 24 cm,
- Inbred strain #2, mean height = 24 cm,
- F1 mean height = 24 cm,
- F2, 12 - 36 cm range, mean = 24 cm,
  - 4/1000 are 12 cm, smallest class
  - 4/1000 are 36 cm, largest class

a. What mode of inheritance?

Quantitative.

b. How many gene pairs?

$$1/4^n = 1/250, n = 4$$

$4/1000 = 1/250$  are the extreme class,

solve  $1/4^n = \text{ratio of extreme class}$

$$4^2 = 16, \quad 4^3 = 64, \quad 4^4 = 256, \text{ etc.}$$

- **Inbred strain #1, mean height = 24 cm,**
- **Inbred strain #2, mean height = 24 cm,**
- **F1 mean height = 24 cm,**
- **F2, 12 - 36 cm range, mean = 24 cm,**
  - **4/1000 are 12 cm, smallest class**
  - **4/1000 are 36 cm, largest class**

**c. How much does each allele contribute?**

**range: largest (36 cm) - smallest (12 cm) = 24 cm**

**alleles: 4 genes, 8 potential additive alleles**

**24 cm / 8 additive alleles = 3 cm / additive allele**



- Inbred strain #1, mean height = 24 cm,
- Inbred strain #2, mean height = 24 cm,
- F1 mean height = 24 cm,
- F2, 12 - 36 cm range, mean = 24 cm,
  - 4/1000 are 12 cm, smallest class
  - 4/1000 are 36 cm, largest class

**d. Indicate one possible set of P1, F1.**

**Base Height is 12 cm, so each P1, F1 has 4 additive alleles.**

**P1    AABBccdd   x   aabbCCDD**

**F1    AaBbCcDd   x   AaBbCcDd**

## Research Article

# Common DNA Variants Accurately Rank an Individual Extreme Height

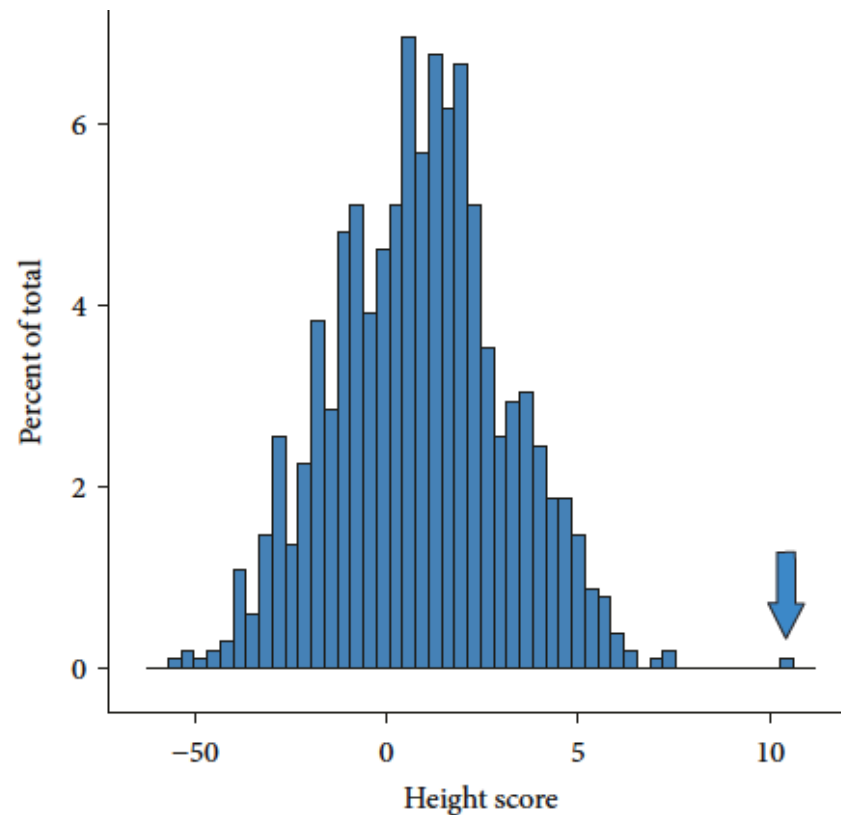


FIGURE 2: Height score distribution calculated using the 2910 SNPs. Mr. Bradley's height score (10.32, indicated by the arrow) ranked highest when compared to the 1020 individuals from ADNI and Cache County, while the next highest was 7.43. The mean height score within the ADNI and Cache County data was 0.98 with a standard deviation of 2.22, making Mr. Bradley's height score 4.2 standard deviations above the mean.

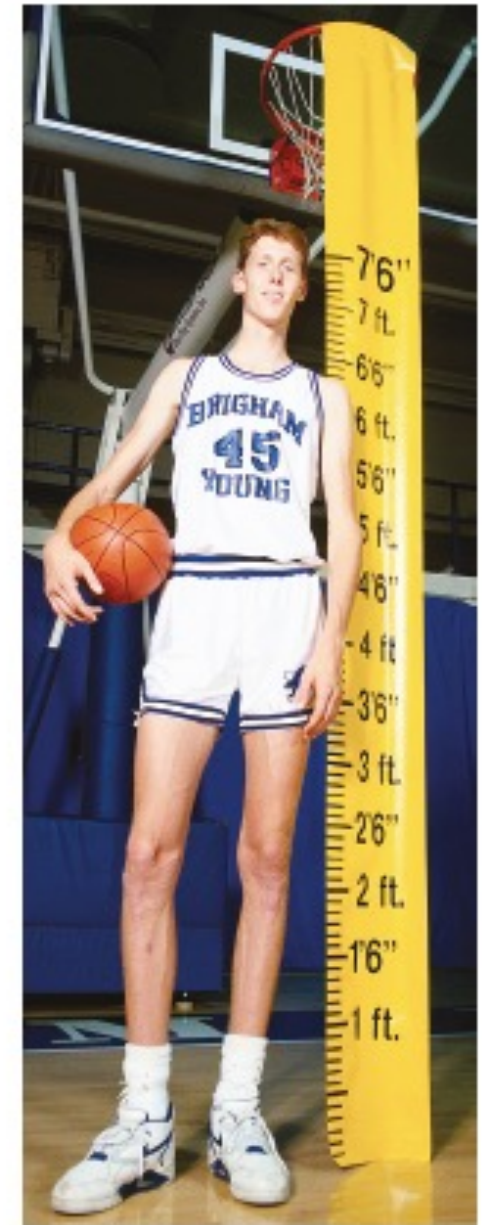
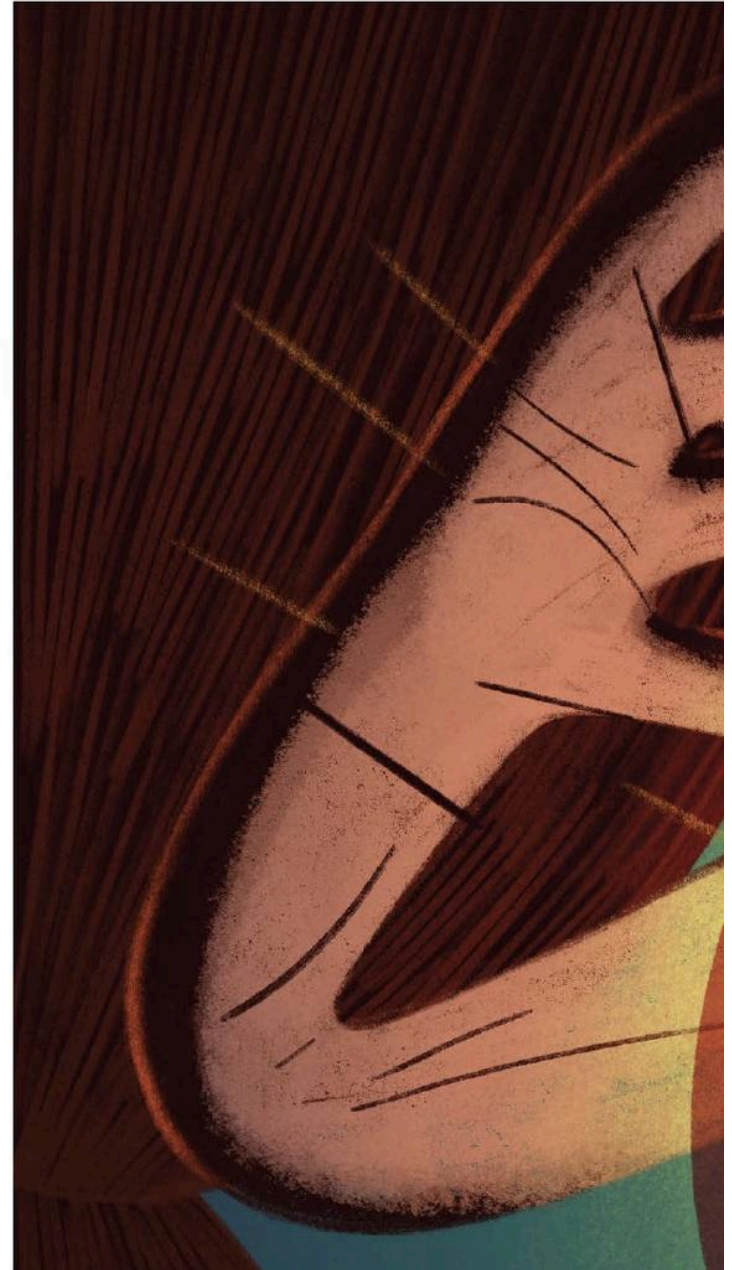


FIGURE 1: Shawn Bradley is 2.29m (7' 6") tall with no known medical conditions. Mr. Bradley played basketball for Brigham Young University from 1990 to 1991. He played in the National Basketball Association from 1993–2005. Photo courtesy of BYU photography.

# THE CONTROVERSIAL EMBRYO TESTS THAT PROMISE A HEALTHIER BABY

---

Some companies offer tests that rank embryos based on their risk of developing complex diseases such as schizophrenia or heart disease. Are the tests accurate — or ethical? **By Max Kozlov**

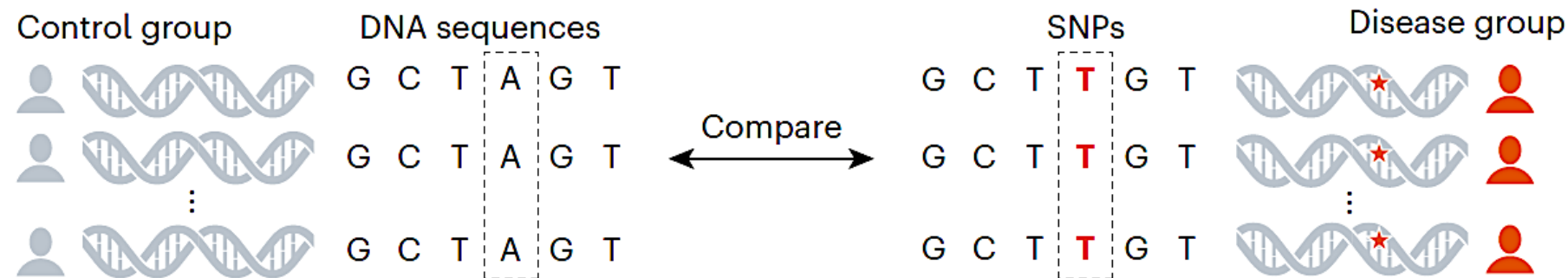


# SCORING EMBRYOS

Some companies are selling genetic tests for embryos generated by *in vitro* fertilization that they say can identify an embryo with the lowest risk of developing common diseases, such as diabetes and some cancers. The method relies on sequencing the embryo's genome and comparing it with existing data on genetic risk factors in adult populations.

## Genetics and risk

Researchers have discovered links between individual DNA letter changes, called single nucleotide polymorphisms (SNPs), and the risk of having a disease. To do this, they compare genome and health data from thousands of people with or without various diseases.

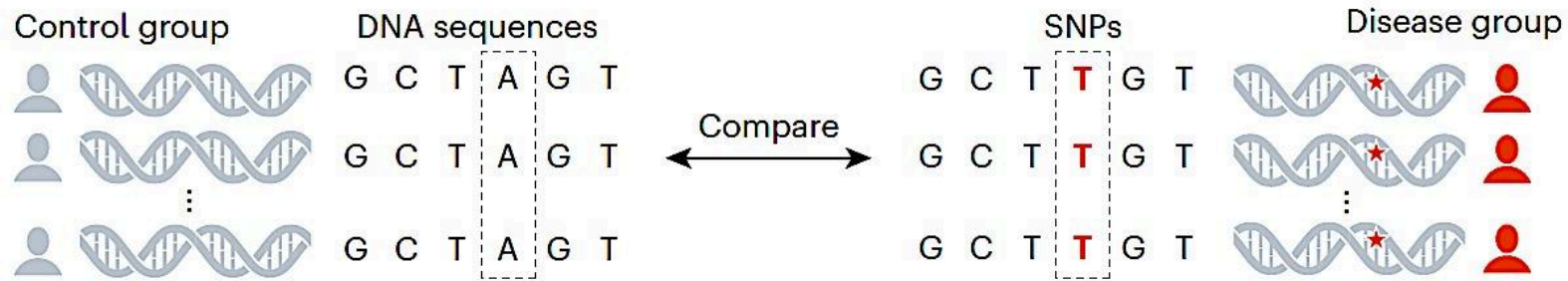


They look across the genome for SNPs that correlate with disease risk.

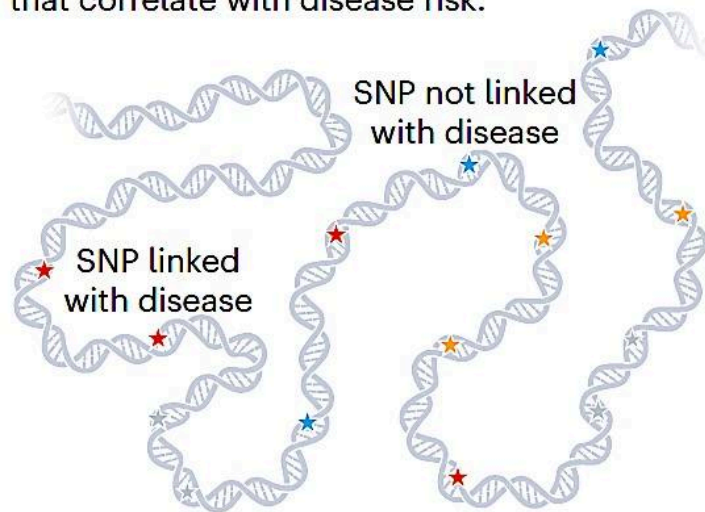


Combining information on all SNPs yields a prediction for how likely someone is to develop a disease.



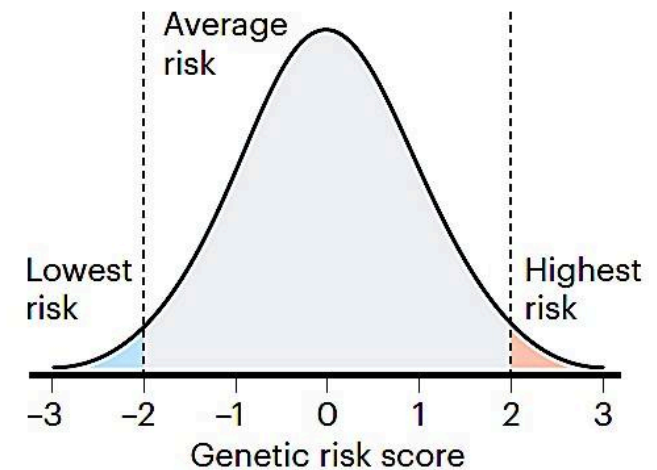


They look across the genome for SNPs that correlate with disease risk.



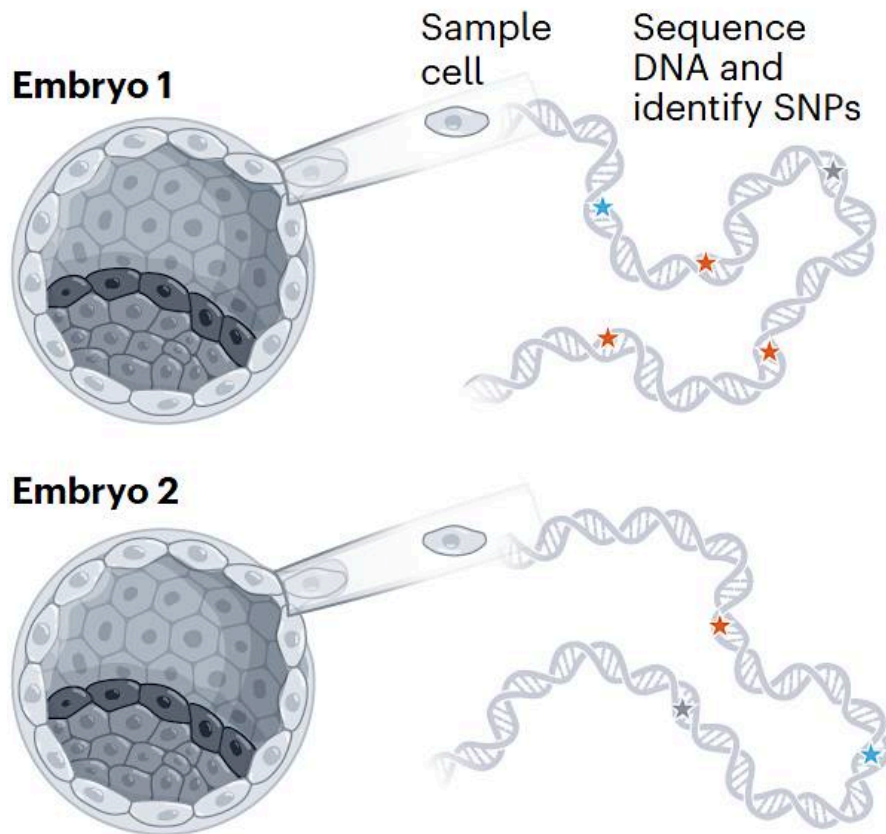
Control for age,  
sex and ancestry

Combining information on all SNPs yields a prediction for how likely someone is to develop a disease.



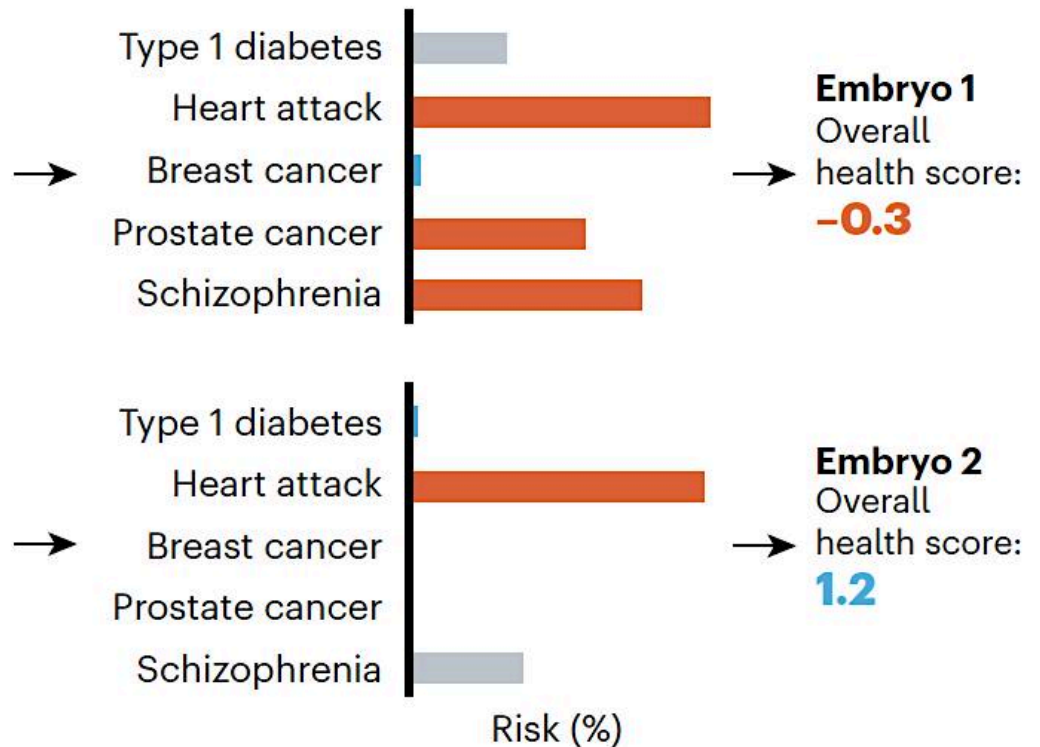
## Embryo sampling

To apply this information to an embryo, clinicians take a few cells from embryos that are about 5 days old. They extract and sequence the DNA and look for SNPs.



## Calculate scores

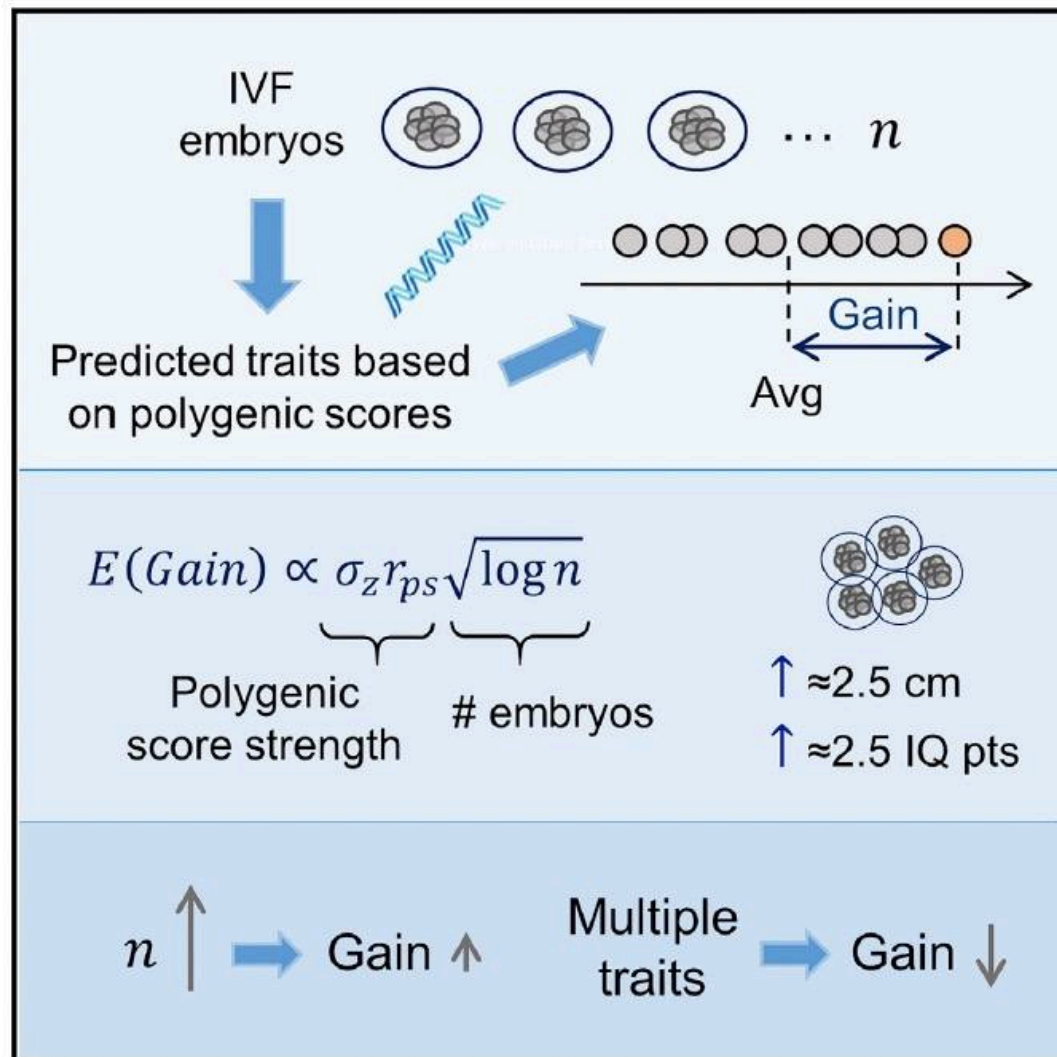
One company, Genomic Prediction, uses this method to help clients to identify embryos with a low risk of developing 12 disorders, including diabetes and some cancers. Each embryo's overall score is calculated by combining the risk of each disease and weighting them by their effect on life expectancy. A higher health score suggests a lower overall risk.





# Screening Human Embryos for Polygenic Traits Has Limited Utility

## Graphical Abstract



## Authors

Ehud Karavani, Or Zuk, Danny Zeevi, ..., Max Lam, Todd Lencz, Shai Carmi

## Correspondence

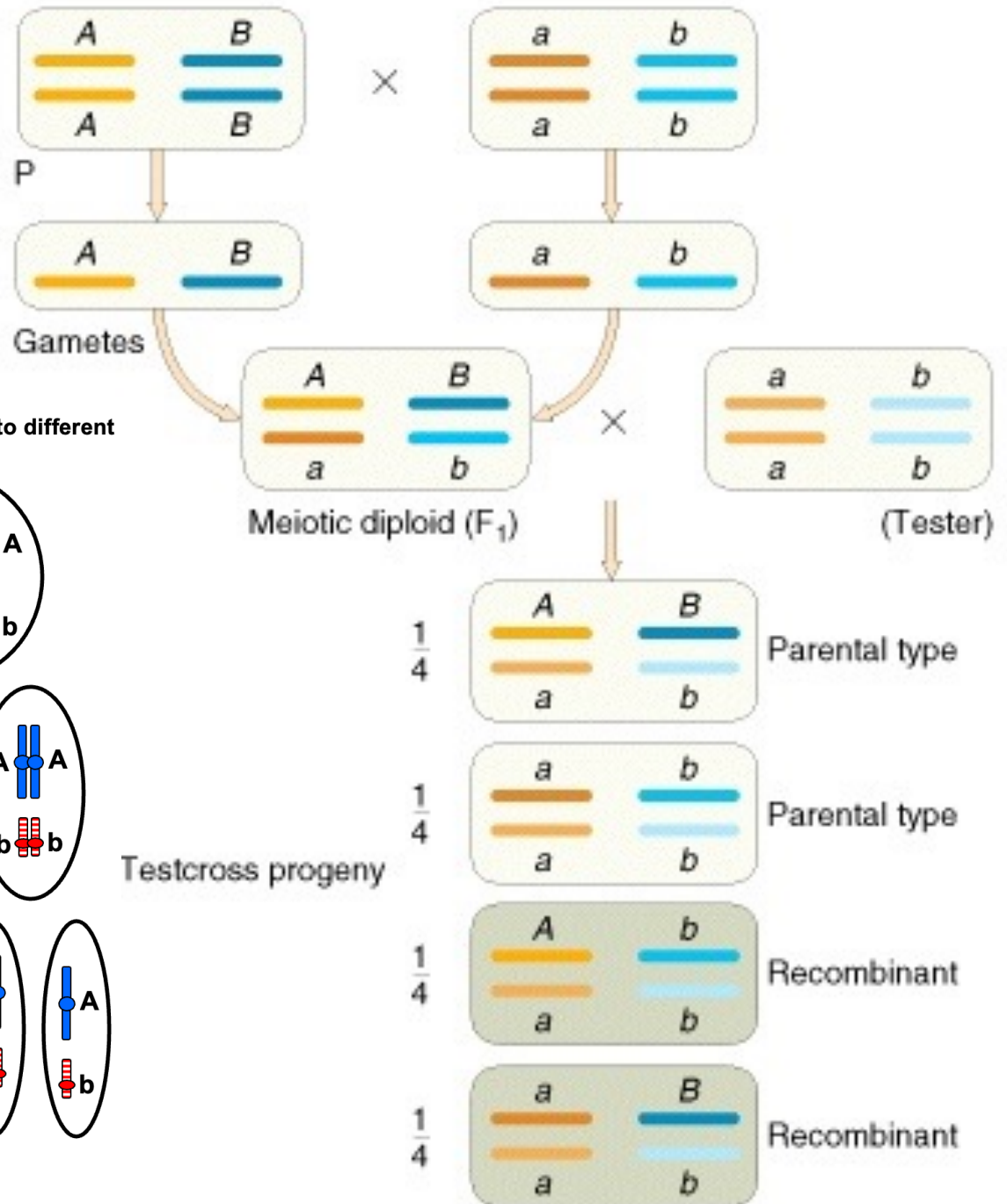
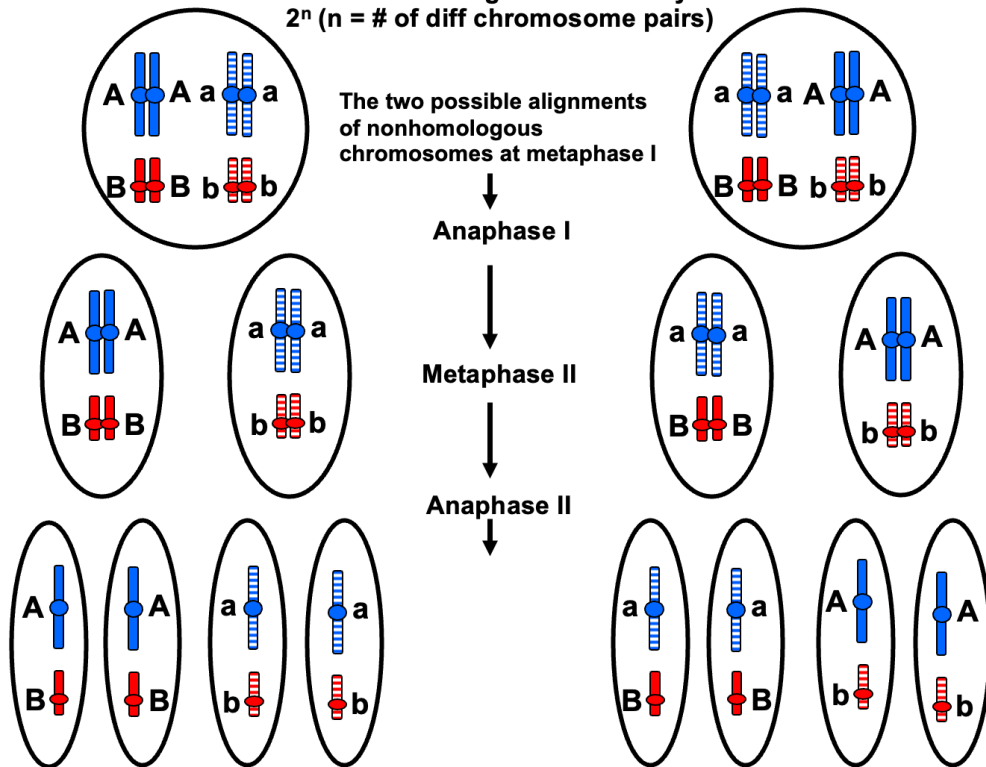
tlencz@northwell.edu (T.L.), shai.carmi@huji.ac.il (S.C.)

## In Brief

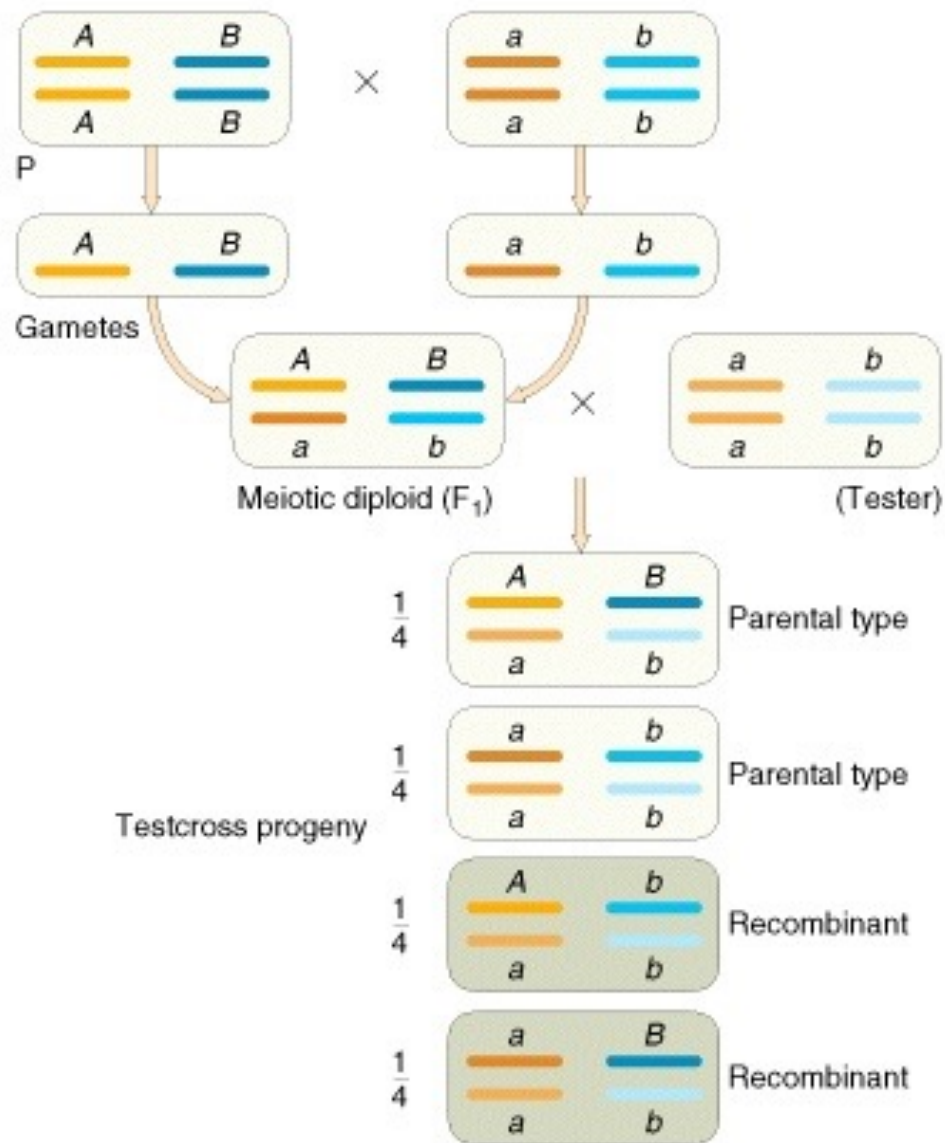
Recent progress in genetic testing of embryos has made it technically feasible to profile IVF embryos for polygenic traits such as height or IQ, but simulations, models, and empirical data show that the gain in trait value when selecting the top-scoring embryo is currently limited and uncertain.

# Independent assortment

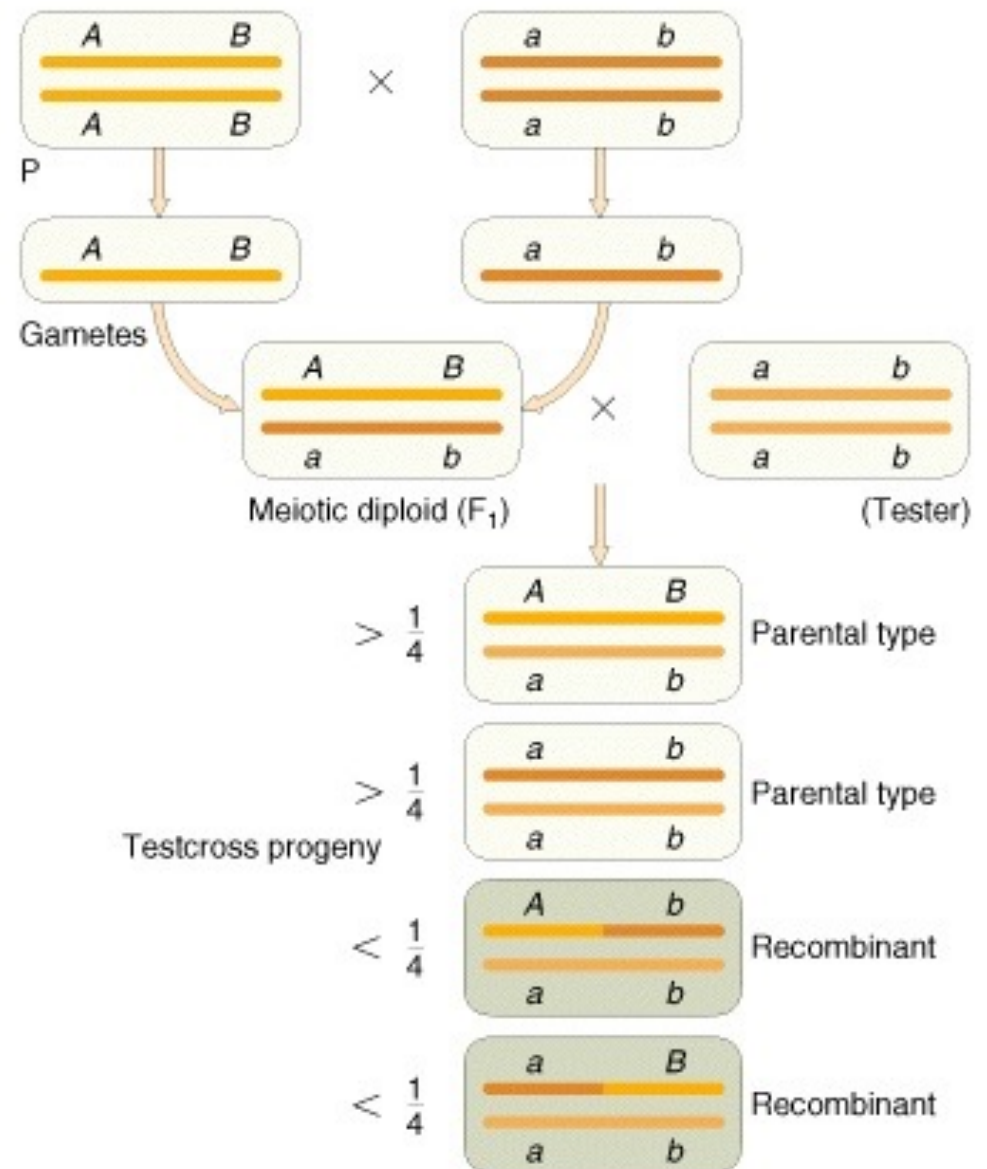
Meiosis without recombination. Independent assortment of homologs to different Poles creates gamete diversity  
 $2^n$  ( $n$  = # of diff chromosome pairs)



# Independent assortment



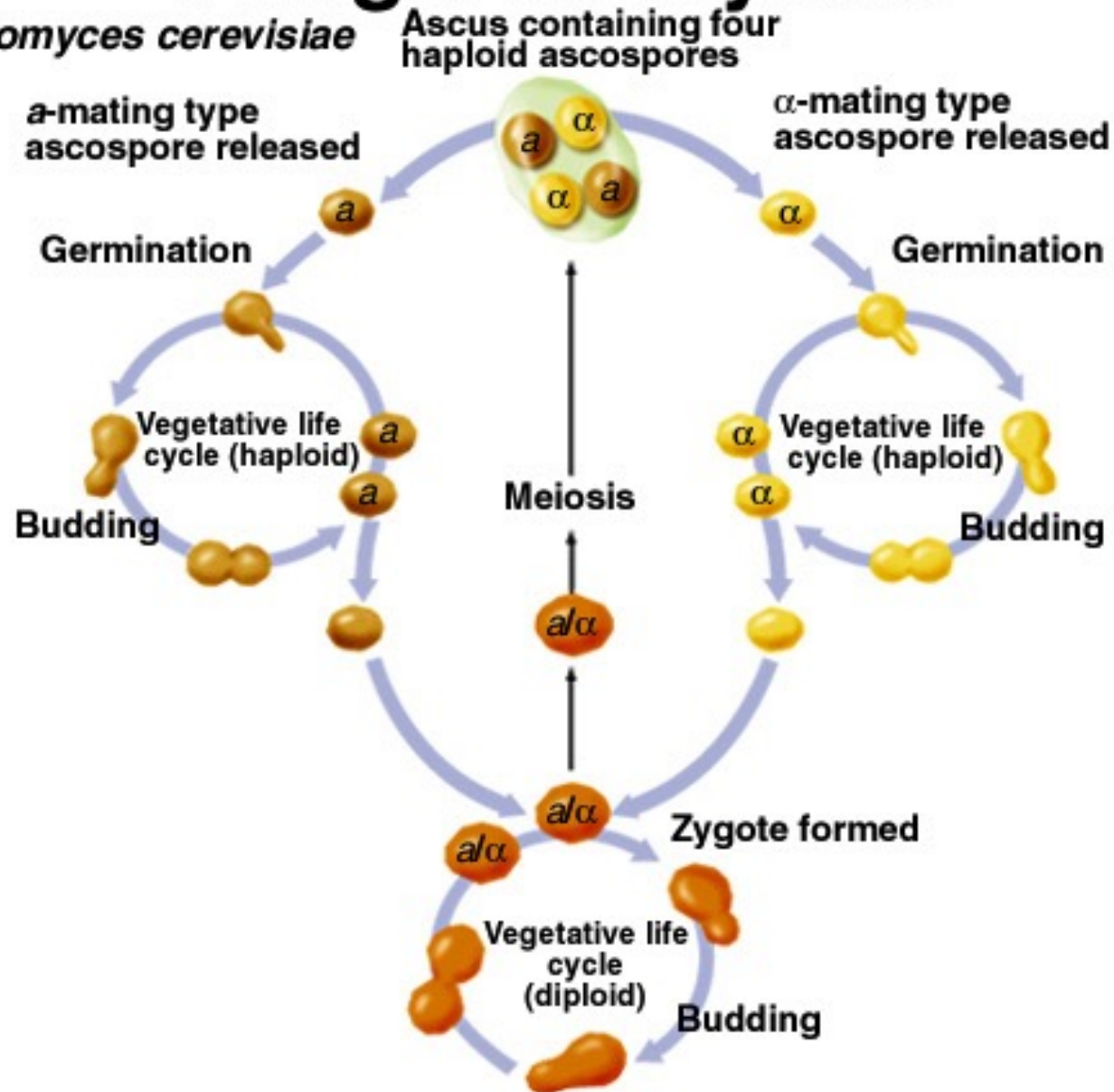
# Linkage



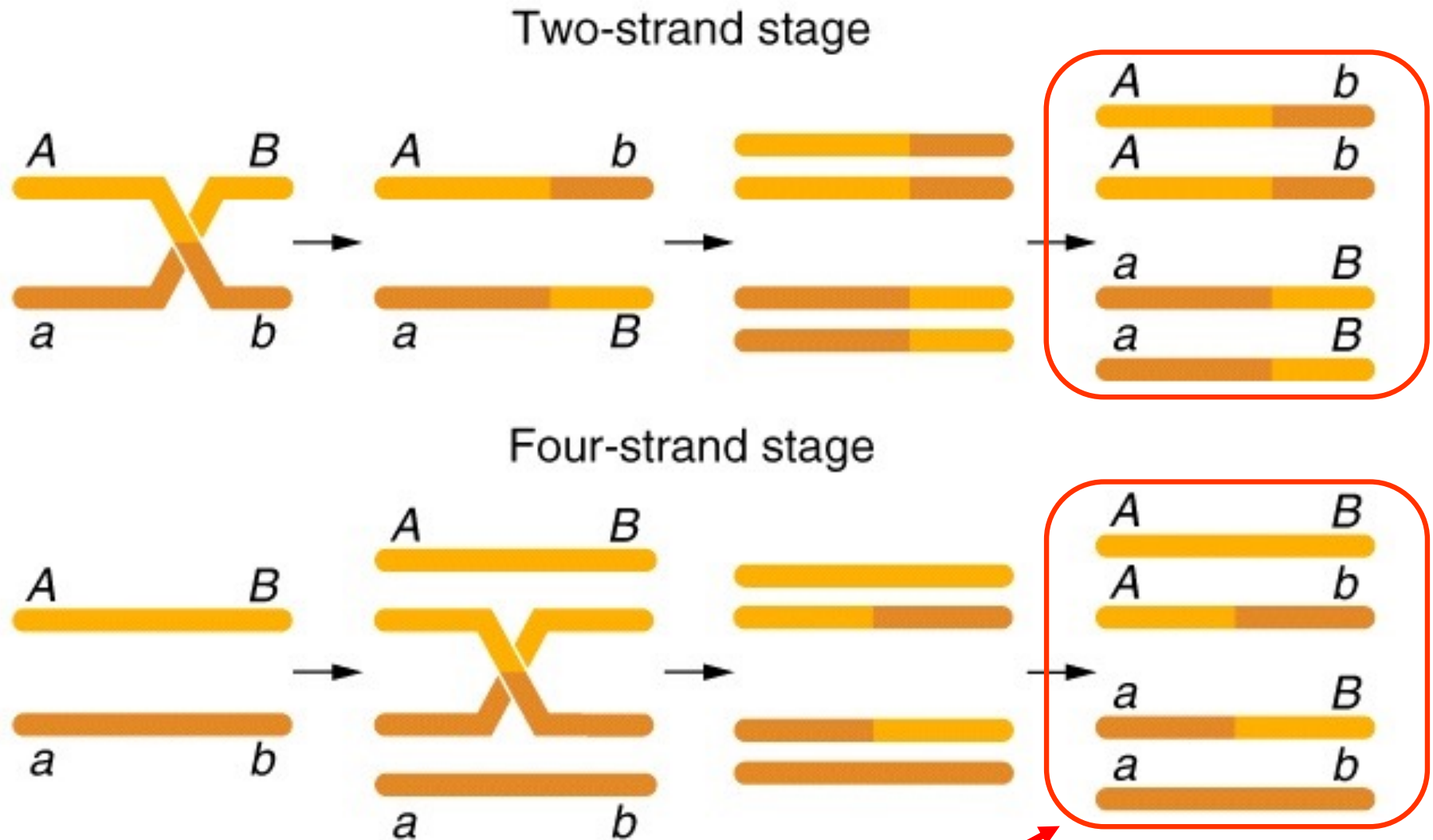


# Fungal life cycles

(a) *Saccharomyces cerevisiae*



# Recombination happens at the 4-strand stage



ascus containing four haploid ascospores (meiotic products)



# Measuring distance between genes.



Alfred Henry Sturtevant

Photo courtesy of Cold Spring Harbor  
Laboratory Archives.

- Alfred Sturtevant, an undergraduate in Morgan's lab proposed that the % recombinants could be used to measure distance between genes.
- Using this approach he constructed first genetic map.

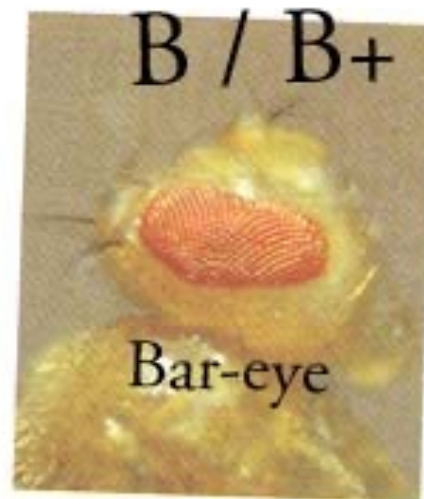
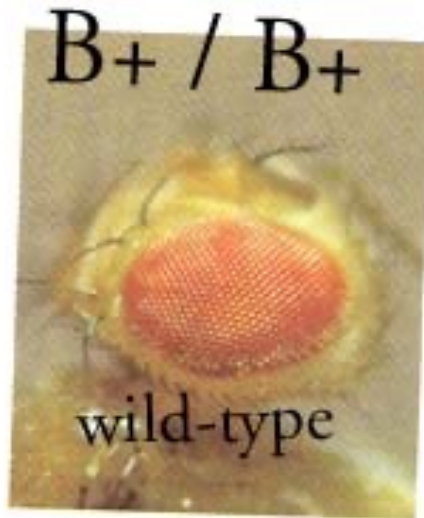


Wild type  
 $\text{white}^+$



$\text{white}^-$

The gene *white* makes a red pigment.



*m* (Miniature wings)

# BUT, the ratio is not 1:1:1:1

Parental phenotypes

$\left( \frac{w}{w} \frac{m}{m} \right)$  *white eyes, miniature wings*, ♀ × *Wild-type* ♂  $\left( \frac{w^+}{w^+} \frac{m^+}{m^+} \right)$

F<sub>1</sub> phenotypes

$\frac{w^+}{w} \frac{m^+}{m}$

*Wild-type* ♀

*white eyes, miniature wings*, ♂

recombinant

F<sub>2</sub> phenotypes



*white eyes, miniature wings*

♀ 359



♂ 391



*Wild-type*

♀ 439



♂ 352

Total: 750

Total: 791

Total parental



*white eyes, wild-type wings*

♀ 218



♂ 237

Total: 455



*Wild-type eyes, miniature wings*

♀ 235



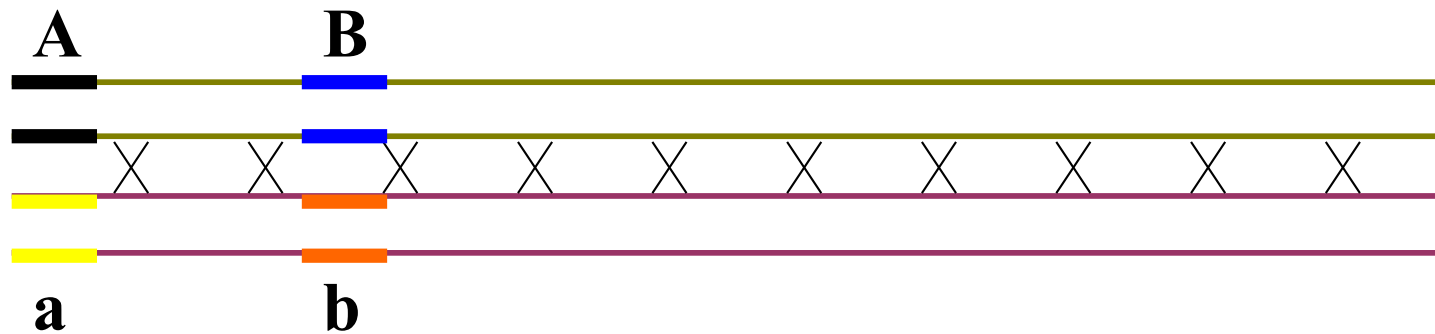
♂ 210

Total: 445

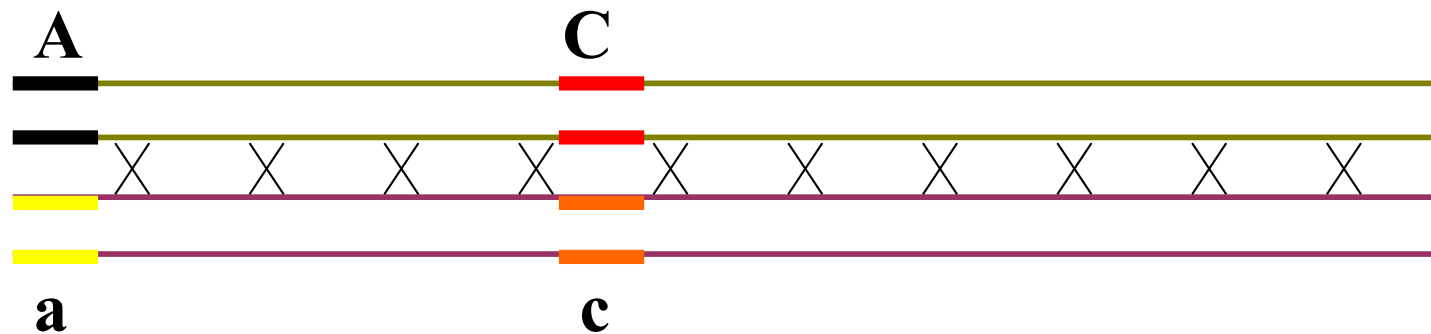
Total recombinant

# **Sturtevant proposed:**

- **Alleles that lie on the same chromosome tend to remain together during meiosis.**
- **Chiasmata (sites of attachment between homologs) were sites of reciprocal exchange between homologs, a process that he referred to as crossing over.**
- **Crossing over produced recombinant gametes.**
- **The closer two genes are, the less likely crossing over will occur between them and fewer recombinants will be produced.**



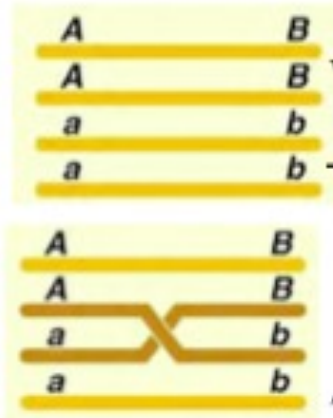
**In this example, there is a 2/10 chance of recombination.**



**In this example, there is a 4/10 chance of recombination.**



# *white (w)* and *miniature (m)* mapping



*wm*

++

*w m* / + + X *w m* / Y



|     |                                           |     |
|-----|-------------------------------------------|-----|
| 770 | red eyes, normal wings (++)               | } P |
| 716 | white eyes, miniature wings ( <i>wm</i> ) |     |
| 401 | red eyes, miniature wings (+ <i>m</i> )   | } R |
| 318 | white eyes, normal wings ( <i>w</i> +)    |     |

$R_f = 401 + 318 / 2205 = 0.326$ ; 32.6% recombination;  
32.6 map units (mu); 32.6 centiMorgans (cM)

# *white (w) and yellow (y) mapping*

$w y / + + \times w y / Y$



|      |                                        |   |          |
|------|----------------------------------------|---|----------|
| 1159 | red eyes, brown body (+ +)             | } | <b>P</b> |
| 1017 | white eyes, yellow body ( <i>w y</i> ) |   |          |
| 12   | red eyes, yellow body (+ <i>y</i> )    | } | <b>R</b> |
| 17   | white eyes, brown body ( <i>w</i> +)   |   |          |

$$Rf = 12 + 17 / 2205 = 0.013 \text{ or } 1.3 \text{ mu}$$

# *yellow (y) and miniature (m) mapping*

*y m* /+ + X *y m* / Y



|            |                                                  |            |
|------------|--------------------------------------------------|------------|
| <b>759</b> | <b>brown body, normal wings (+ +)</b>            | <b>} P</b> |
| <b>700</b> | <b>yellow body, miniature wings (<i>y m</i>)</b> |            |
| <b>417</b> | <b>brown body, miniature wings (+ <i>m</i>)</b>  | <b>} R</b> |
| <b>329</b> | <b>yellow body, normal wings (<i>y</i> +)</b>    |            |

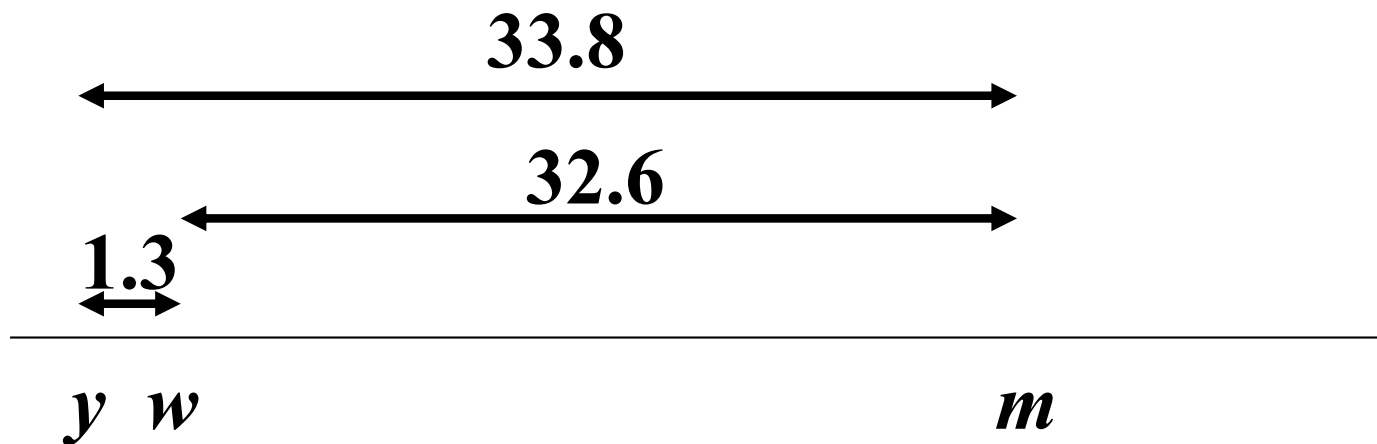
$$\text{Rf} = 417 + 329 / 2205 = 0.338 \text{ or } 33.8 \text{ mu}$$

# Genetic map

$y - w$  1.3 mu

$w - m$  32.6 mu

$y - m$  33.8 mu



Results are consistent with the gene order:  $y w m$

$$\text{Genetic map distance (recombination frequency)} = \frac{\text{\# of recombinant gametes}}{\text{Total gametes}} \times 100$$

Genetic map distance measured in centiMorgans (cM)

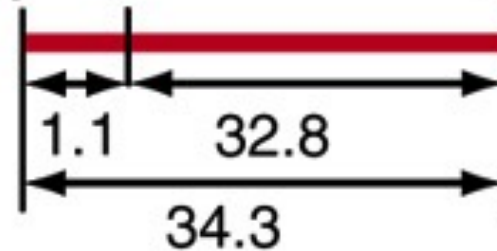


# The first genetic map

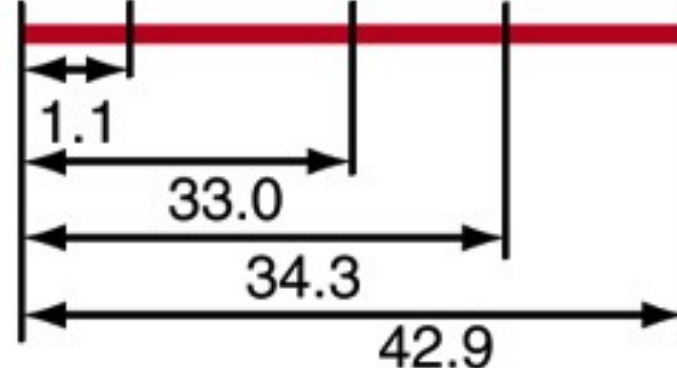
(a)

|            |      |
|------------|------|
| <i>y-w</i> | 1.1  |
| <i>y-v</i> | 33.0 |
| <i>y-m</i> | 34.3 |
| <i>y-r</i> | 42.9 |
| <i>w-v</i> | 32.1 |
| <i>w-m</i> | 32.8 |
| <i>w-r</i> | 42.1 |
| <i>v-m</i> | 4.0  |
| <i>v-r</i> | 24.1 |
| <i>m-r</i> | 17.8 |

(b) *y* *w* *m*



(c) *y* *w* *v* *m* *r*



(d) *y* *w* *v* *m* *r*



# A 3-point cross

(a)

P

♀ *vg b pr* / *vg b pr* × ♂ *vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup>* / *vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup>*

F<sub>1</sub> (all identical)

*vg b pr* / *vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup>*

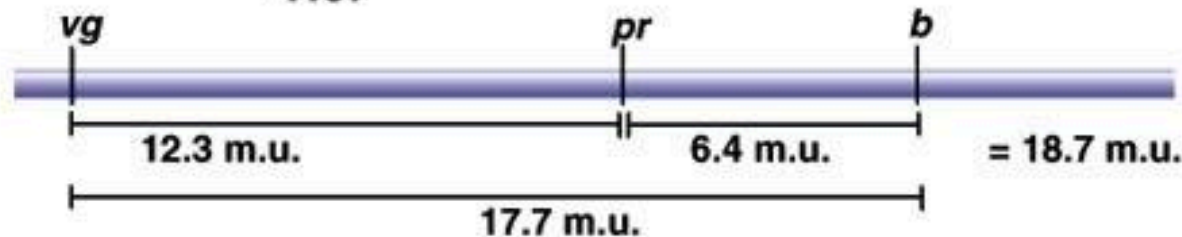
Test cross

♀ *vg b pr* / *vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup>* × ♂ *vg b pr* / *vg b pr*

Test cross progeny

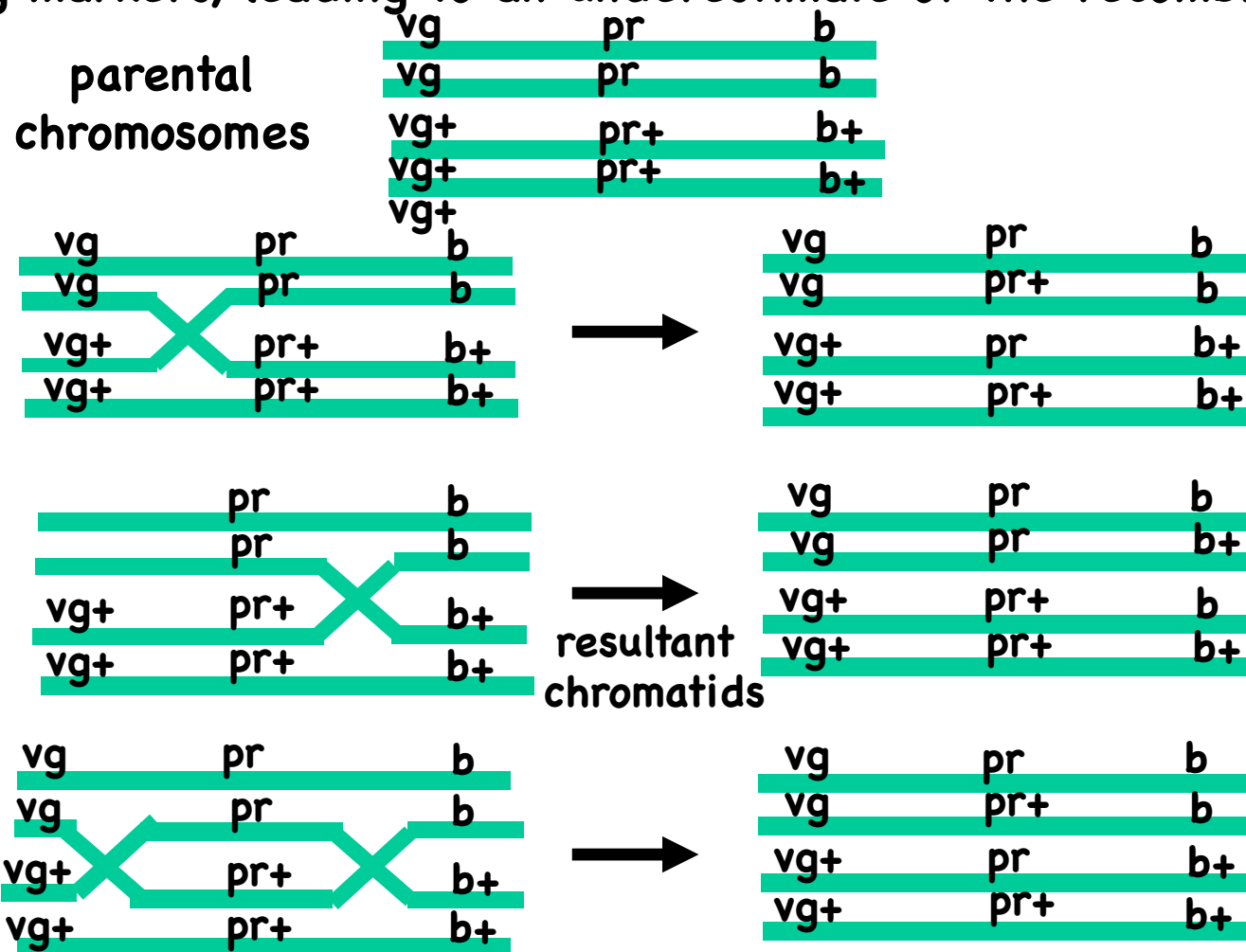
|             |                                                    |                                                                                           |
|-------------|----------------------------------------------------|-------------------------------------------------------------------------------------------|
| 1779        | <i>vg b pr</i>                                     | } Parental combinations for all three genes                                               |
| 1654        | <i>vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup></i> |                                                                                           |
| 252         | <i>vg<sup>+</sup> b pr</i>                         | } Recombinants for <i>vg</i> relative to parental combinations for <i>b</i> and <i>pr</i> |
| 241         | <i>vg b<sup>+</sup> pr<sup>+</sup></i>             |                                                                                           |
| 131         | <i>vg<sup>+</sup> b pr<sup>+</sup></i>             | } Recombinants for <i>b</i> relative to parental combinations for <i>vg</i> and <i>pr</i> |
| 118         | <i>vg b<sup>+</sup> pr</i>                         |                                                                                           |
| 13          | <i>vg b pr<sup>+</sup></i>                         | } Recombinants for <i>pr</i> relative to parental combinations for <i>vg</i> and <i>b</i> |
| 9           | <i>vg<sup>+</sup> b<sup>+</sup> pr</i>             |                                                                                           |
| <u>4197</u> |                                                    |                                                                                           |

(b)



$$\frac{252 + 241 + 131 + 118}{4197} \times 100 = \text{vg-b} = 17.7 \text{ m. u.}$$

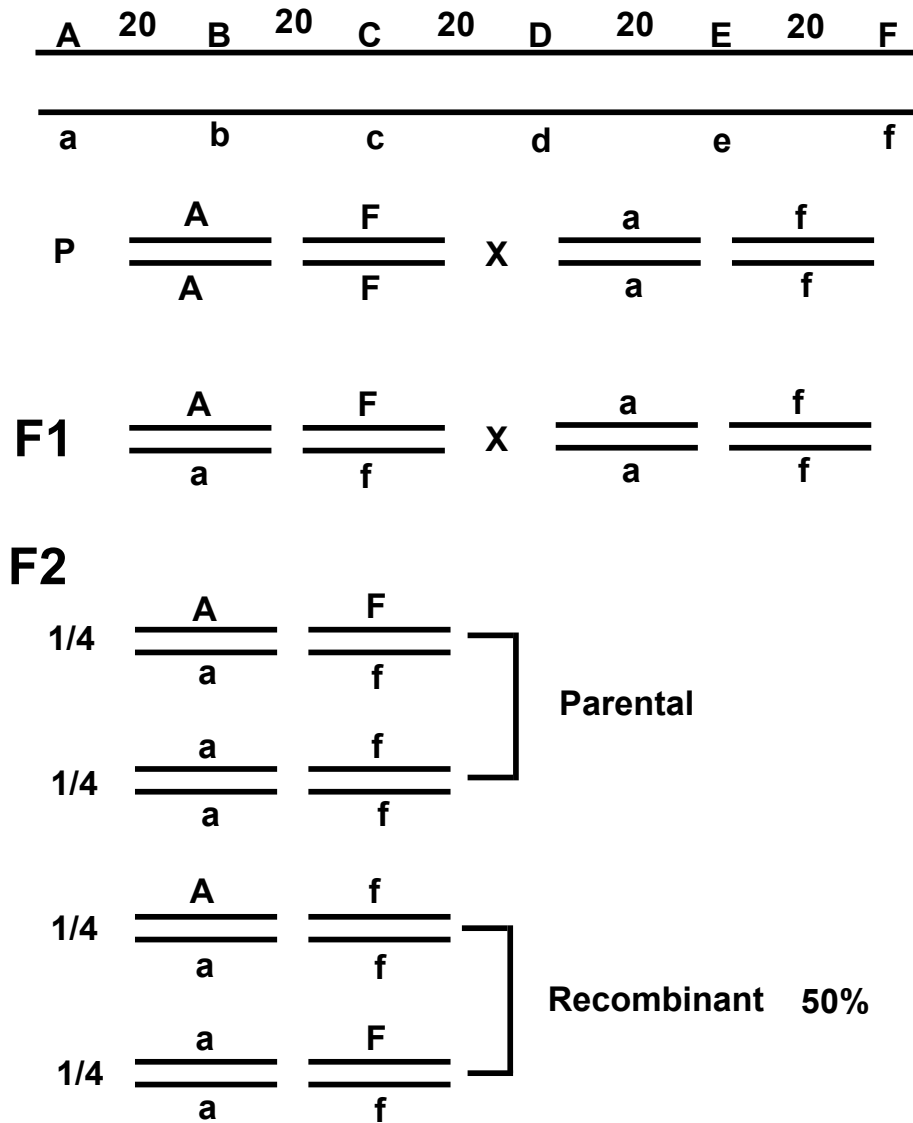
Double crossovers between two markers can result in parental genotypes for flanking markers, leading to an underestimate of the recombination distance.



These double crossovers can be identified if there are markers (alleles of genes) in the middle (in this case pr, pr+) whose recombination can be followed.

$$\frac{252 + 241 + 131 + 118 + \boxed{13 + 13 + 9 + 9}}{4197}$$

The largest map distance that can be measured between 2 genes  
is 50 map units



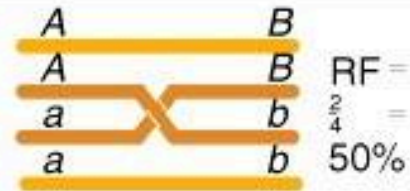
Genes far away on a chromosome act as if they are unlinked

### No crossovers



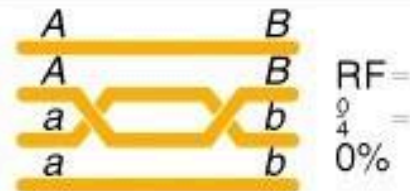
### One crossover

(Can be between any nonsister pair.)



### Two crossovers

(Holding one crossover constant and varying the position of the second produces four equally frequent two-crossover meioses.)



Two-strand double crossover



Three-strand double crossover



Three-strand double crossover



Four-strand double crossover

**One or more crossovers looks, on average, like 50% recombination.**

$$\text{Average two-crossover RF} = \frac{8}{16} = 50\%$$

# Interference

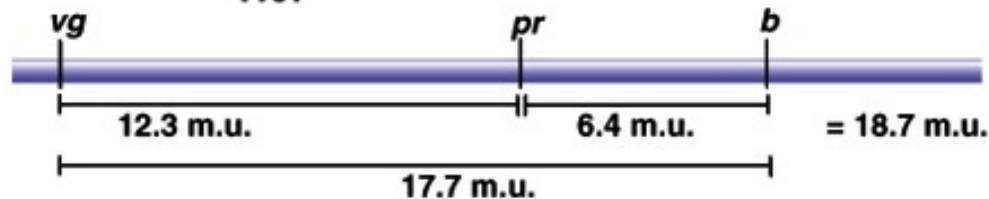
Test cross

♀ *vg b pr / vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup>* × ♂ *vg b pr / vg b pr*

Test cross progeny

|      |                                                    |                                                                                         |
|------|----------------------------------------------------|-----------------------------------------------------------------------------------------|
| 1779 | <i>vg b pr</i>                                     | Parental combinations for all three genes                                               |
| 1654 | <i>vg<sup>+</sup> b<sup>+</sup> pr<sup>+</sup></i> |                                                                                         |
| 252  | <i>vg<sup>+</sup> b pr</i>                         | Recombinants for <i>vg</i> relative to parental combinations for <i>b</i> and <i>pr</i> |
| 241  | <i>vg b<sup>+</sup> pr<sup>+</sup></i>             |                                                                                         |
| 131  | <i>vg<sup>+</sup> b pr<sup>+</sup></i>             | Recombinants for <i>b</i> relative to parental combinations for <i>vg</i> and <i>pr</i> |
| 118  | <i>vg b<sup>+</sup> pr</i>                         |                                                                                         |
| 13   | <i>vg b pr<sup>+</sup></i>                         | Recombinants for <i>pr</i> relative to parental combinations for <i>vg</i> and <i>b</i> |
| 9    | <i>vg<sup>+</sup> b<sup>+</sup> pr</i>             |                                                                                         |
| 4197 |                                                    |                                                                                         |

(b)



If the probability of one recombinant in a given interval (*vg*--*b*) is *p* then the probability of two such events is *p* X *p*. But this is not always the case.

$$\begin{aligned} b\text{--}pr &= 0.064 \\ vg\text{--}pr &= 0.123 \end{aligned}$$

$$0.123 \times 0.064 = .0079 = .79\% \text{ Expected}$$

$$\frac{13 + 9}{4197} = 0.52\% \text{ Observed}$$

The occurrence of one crossover reduces the likelihood of a second crossover  
= interference

$$\text{coefficient of coincidence} = \frac{\text{Frequency of observed doubles}}{\text{Frequency of expected doubles}} = \frac{0.52}{0.79} = 0.66$$

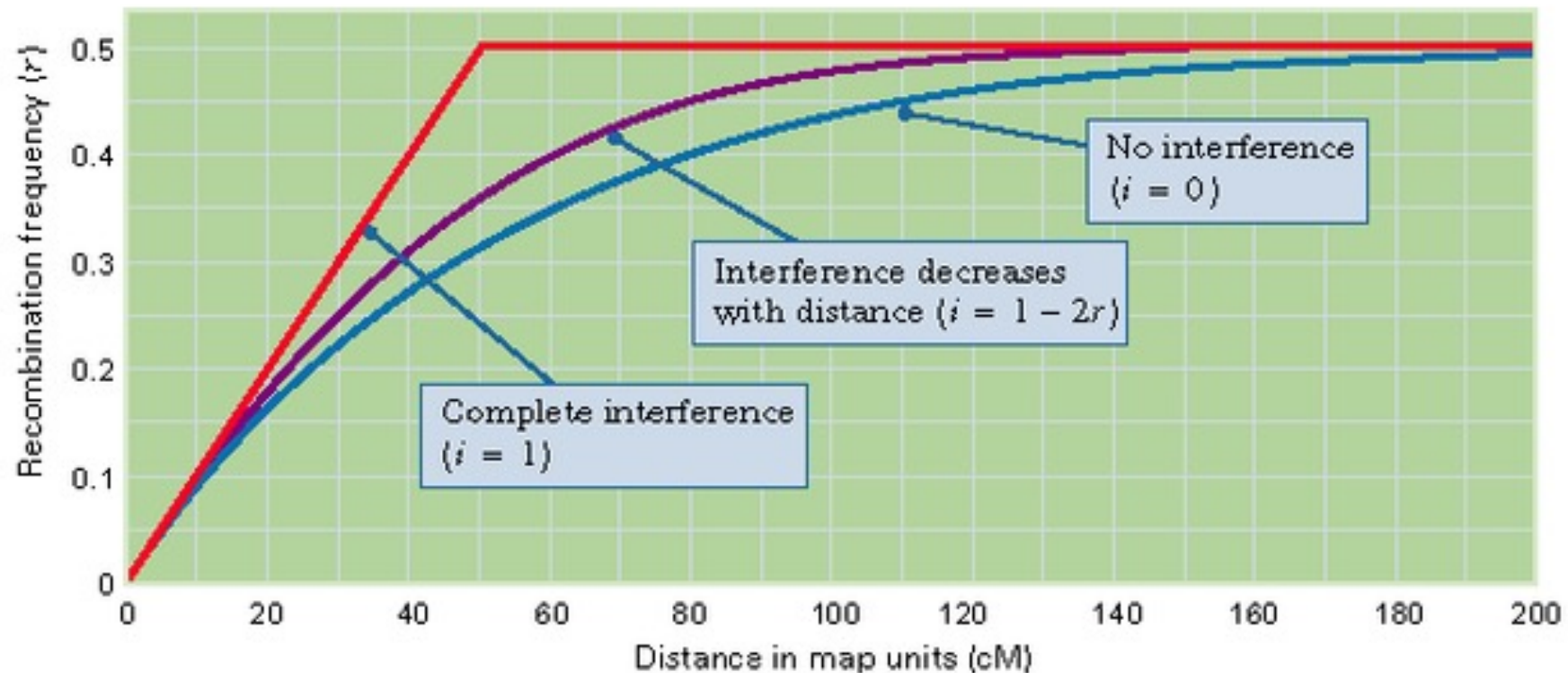
Interference = 1 - coefficient of coinc.

*I* = 0 = crossovers do not interfere with each other

*I* = 1 = There are no double crossovers

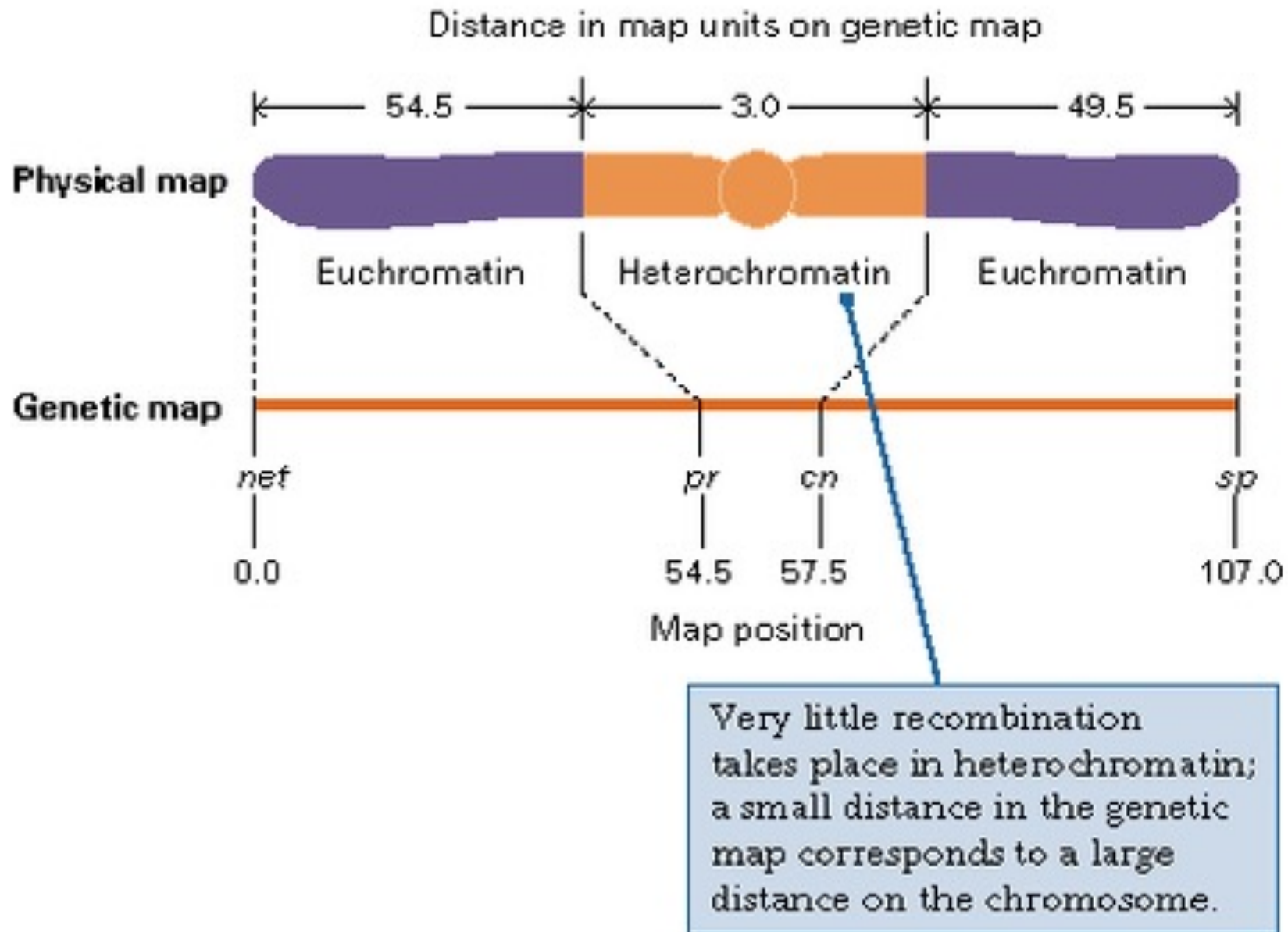


**When interference is low we need some method for correcting for the presence of multiple crossovers in order to generate an accurate genetic map**



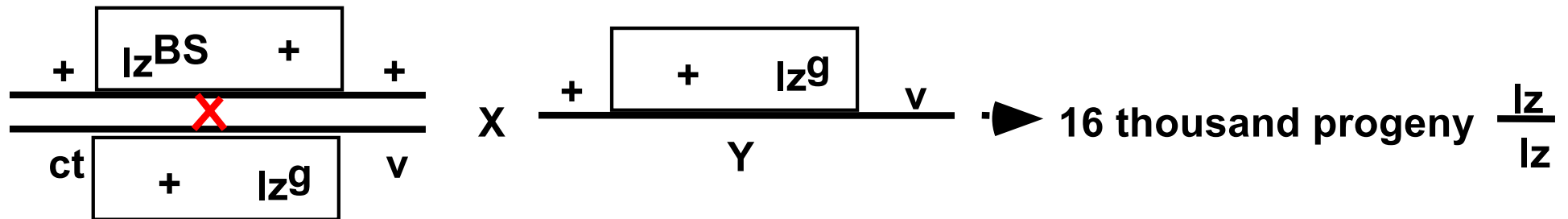
**Bottom line: best to measure distances over a series of small distances rather than over one large distance**

## Map units/physical distance depends on the nature of the chromatin involved



# The unit of recombination is the basepair

Therefore, recombination can also occur within a gene

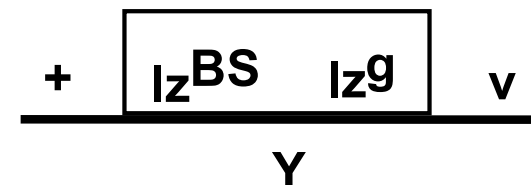


134 males and females *lz*<sup>+</sup> =  $8 \times 10^{-3}$

Reversion rate =  $10^{-6}$



Only a few of the reciprocal recombinants (5)  
They are semilethal.



# Chi-Square Test

- A fundamental problem in genetics is determining whether the experimentally determined data fits the results expected from theory (i.e. Mendel's laws as expressed in the Punnett square).
- How can you tell if an observed set of offspring counts is legitimately the result of a given underlying simple ratio? For example, you do a cross and see 290 purple flowers and 110 white flowers in the offspring. This is pretty close to a  $3/4 : 1/4$  ratio, but how do you formally define "pretty close"? What about 250:150?

# Goodness of Fit

- Mendel had no way of solving this problem. Shortly after the rediscovery of his work in 1900, Karl Pearson and R.A. Fisher developed the “chi-square” test for this purpose.
- The chi-square test is a “goodness of fit” test: it answers the question of how well do experimental data fit expectations.
- We start with a theory for how the offspring will be distributed: the “null hypothesis”. We will discuss the offspring of a self-pollination of a heterozygote. The null hypothesis is that the offspring will appear in a ratio of 3/4 dominant to 1/4 recessive.

# Formula

- First determine the number of each phenotype that have been observed and how many would be expected given basic genetic theory.
- Then calculate the chi-square statistic using this formula.
- The “X” is the Greek letter chi; the “ $\sum$ ” is a sigma; it means to sum the following terms for all phenotypes. “obs” is the number of individuals of the given phenotype observed; “exp” is the number of that phenotype expected from the null hypothesis.
- Note that you must use the number of individuals, the counts, and NOT proportions, ratios, or frequencies.

$$X^2 = \sum \frac{(obs - exp)^2}{exp}$$

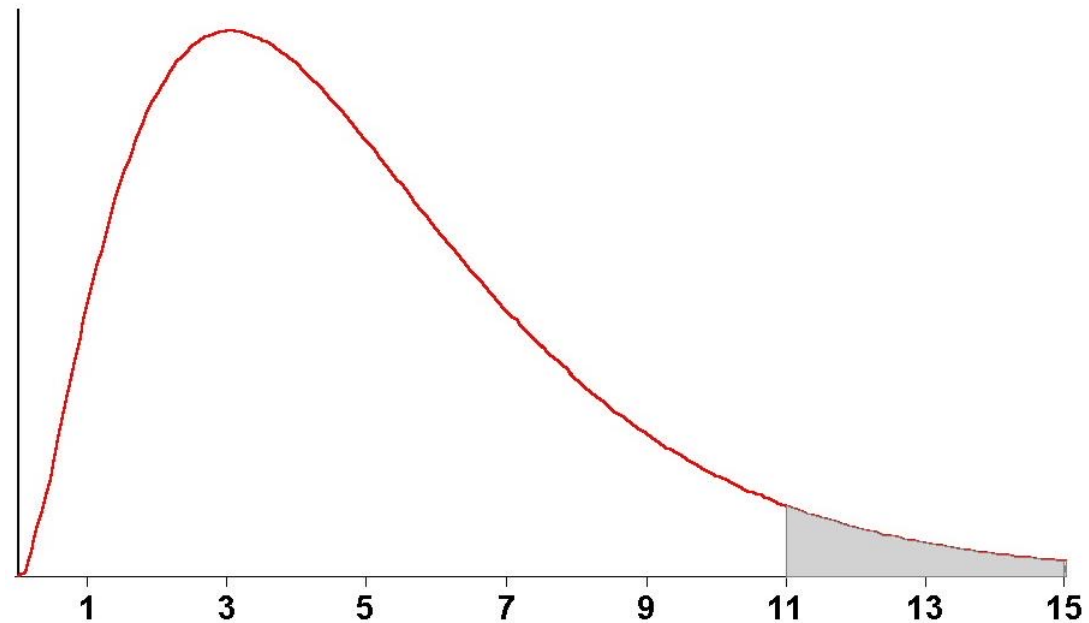


# Example

- As an example, you count F2 offspring, and get 290 purple and 110 white flowers. This is a total of 400 (290 + 110) offspring.
- We expect a 3/4 : 1/4 ratio. We need to calculate the expected numbers (you MUST use the numbers of offspring, NOT the proportion!!!); this is done by multiplying the total offspring by the expected proportions. This we expect  $400 * 3/4 = 300$  purple, and  $400 * 1/4 = 100$  white.
- Thus, for purple, obs = 290 and exp = 300. For white, obs = 110 and exp = 100.
- Now it's just a matter of plugging into the formula:
$$\begin{aligned}X^2 &= (290 - 300)^2 / 300 + (110 - 100)^2 / 100 \\&= (-10)^2 / 300 + (10)^2 / 100 \\&= 100 / 300 + 100 / 100 \\&= 0.333 + 1.000 \\&= 1.333.\end{aligned}$$
- This is our chi-square value: now we need to see what it means and how to use it.

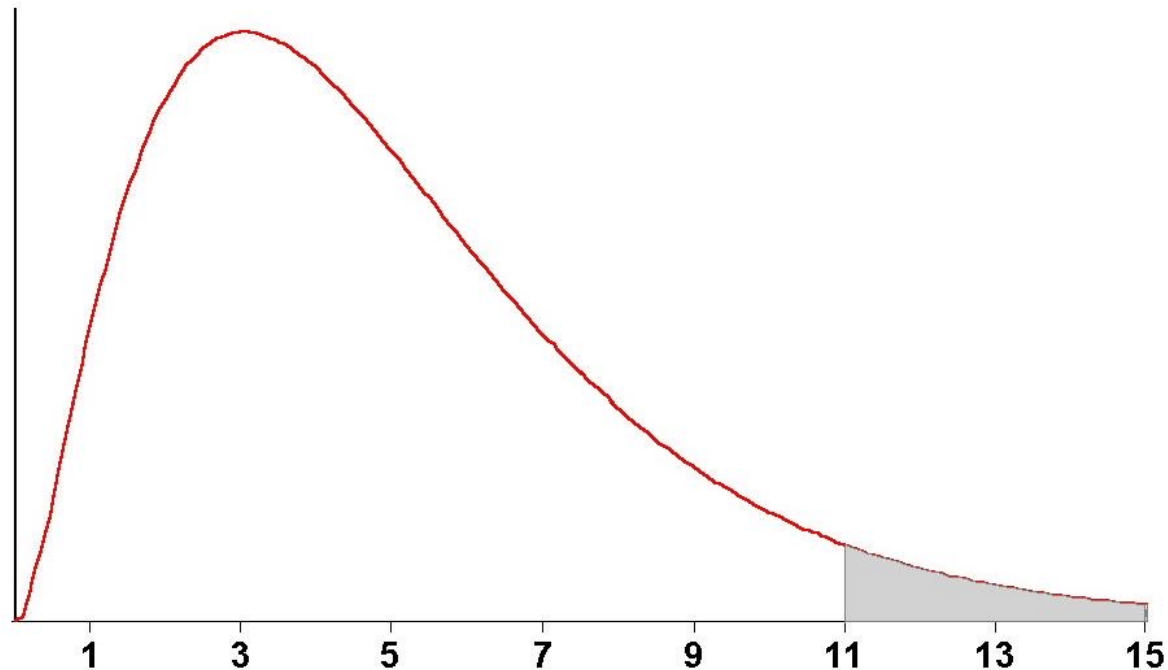
# Chi-Square Distribution

- Although the chi-square distribution can be derived through math theory, we can also get it experimentally:
- Let's say we do the same experiment 1000 times, do the same self-pollination of a Pp heterozygote, which should give the 3/4 : 1/4 ratio. For each experiment we calculate the chi-square value, then plot them all on a graph.
- The x-axis is the chi-square value calculated from the formula. The y-axis is the number of individual experiments that got that chi-square value.



# Chi-Square Distribution, p. 2

- You see that there is a range here: if the results were perfect you get a chi-square value of 0 (because  $\text{obs} = \text{exp}$ ). This rarely happens: most experiments give a small chi-square value (the hump in the graph).
- Note that all the values are greater than 0: that's because we squared the  $(\text{obs} - \text{exp})$  term: squaring always gives a non-negative number.
- Sometimes you get really wild results, with obs very different from exp: the long tail on the graph. Really odd things occasionally do happen by chance alone (for instance, you might win the lottery).



# The Critical Question

- how do you tell a really odd but correct result from a **WRONG** result? The graph is what happens with real experiments: most of the time the results fit expectations pretty well, but occasionally very skewed distributions of data occur even though you performed the experiment correctly, based on the correct theory.
- The simple answer is: you can never tell for certain that a given result is “wrong”, that the result you got was completely impossible based on the theory you used. All we can do is determine whether a given result is likely or unlikely.
- Key point: There are 2 ways of getting a high chi-square value: an unusual result from the correct theory, or a result from the wrong theory. These are indistinguishable; because of this fact, statistics is never able to discriminate between true and false with 100% certainty.
- Using the example here, how can you tell if your 290: 110 offspring ratio really fits a  $3/4 : 1/4$  ratio (as expected from selfing a heterozygote) or whether it was the result of a mistake or accident-- a  $1/2 : 1/2$  ratio from a backcross for example? You can't be certain, but you can at least determine whether your result is reasonable.

# Reasonable

- What is a “reasonable” result is subjective and arbitrary.
- For most work (and for the purposes of this class), a result is said to not differ significantly from expectations if it could happen at least 1 time in 20. That is, if the difference between the observed results and the expected results is small enough that it would be seen at least 1 time in 20 over thousands of experiments, we “fail to reject” the null hypothesis.
- For technical reasons, we use “fail to reject” instead of “accept”.
- “1 time in 20” can be written as a probability value  $p = 0.05$ , because  $1/20 = 0.05$ .
- Another way of putting this. If your experimental results are worse than 95% of all similar results, they get rejected because you may have used an incorrect null hypothesis.

# Degrees of Freedom

- A critical factor in using the chi-square test is the “degrees of freedom”, which is essentially the number of independent random variables involved.
- Degrees of freedom is simply the number of classes of offspring minus 1.
- For our example, there are 2 classes of offspring: purple and white. Thus, degrees of freedom (d.f.) =  $2 - 1 = 1$ .



# Critical Chi-Square

- Critical values for chi-square are found on tables, sorted by degrees of freedom and probability levels. Be sure to use  $p = 0.05$ .
- If your calculated chi-square value is greater than the critical value from the table, you “reject the null hypothesis”.
- If your chi-square value is less than the critical value, you “fail to reject” the null hypothesis (that is, you accept that your genetic theory about the expected ratio is correct).

# Chi-Square Table

**Table 5-2**  
**Critical Values of the  $\chi^2$  Distribution**

| df \ $p$ | 0.995 | 0.975 | 0.9   | 0.5    | 0.1    | 0.05   | 0.025  | 0.01   | 0.005  | df |
|----------|-------|-------|-------|--------|--------|--------|--------|--------|--------|----|
| 1        | .000  | .000  | 0.016 | 0.455  | 2.706  | 3.841  | 5.024  | 6.635  | 7.879  | 1  |
| 2        | 0.010 | 0.051 | 0.211 | 1.386  | 4.605  | 5.991  | 7.378  | 9.210  | 10.597 | 2  |
| 3        | 0.072 | 0.216 | 0.584 | 2.366  | 6.251  | 7.815  | 9.348  | 11.345 | 12.838 | 3  |
| 4        | 0.207 | 0.484 | 1.064 | 3.357  | 7.779  | 9.488  | 11.143 | 13.277 | 14.860 | 4  |
| 5        | 0.412 | 0.831 | 1.610 | 4.351  | 9.236  | 11.070 | 12.832 | 15.086 | 16.750 | 5  |
| 6        | 0.676 | 1.237 | 2.204 | 5.348  | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 6  |
| 7        | 0.989 | 1.690 | 2.833 | 6.346  | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 7  |
| 8        | 1.344 | 2.180 | 3.490 | 7.344  | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 8  |
| 9        | 1.735 | 2.700 | 4.168 | 8.343  | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 9  |
| 10       | 2.156 | 3.247 | 4.865 | 9.342  | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 10 |
| 11       | 2.603 | 3.816 | 5.578 | 10.341 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 11 |
| 12       | 3.074 | 4.404 | 6.304 | 11.340 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 12 |
| 13       | 3.565 | 5.009 | 7.042 | 12.340 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 13 |
| 14       | 4.075 | 5.629 | 7.790 | 13.339 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 14 |
| 15       | 4.601 | 6.262 | 8.547 | 14.339 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 15 |

# Linkage?

- T. H. Morgan crossed *Drosophila melanogaster* wildtype flies with pure breeding purple-eyed, vestigial winged flies
- He then crossed  $F_1$  females heterozygous for purple-eyes (pr) and vestigial wings (vg) with tester males homozygous recessive for pr and vg
- i.e.  $pr^+/pr \cdot vg^+/vg \times pr/pr \cdot vg/vg$

## F1 of testcross

| Class                           | Phenotype                      | O    | Type        |
|---------------------------------|--------------------------------|------|-------------|
| pr <sup>+</sup> vg <sup>+</sup> | Wildtype eyes and wings        | 1339 | Parental    |
| pr vg                           | Purple eyes, vestigial wings   | 1195 | Parental    |
| pr <sup>+</sup> vg              | Wildtype eyes, vestigial wings | 151  | Recombinant |
| pr vg <sup>+</sup>              | Purple eyes, wildtype wings    | 154  | Recombinant |
| Total                           |                                | 2839 |             |

# Linked

- Parental vs recombinant
- 1339+1195 vs 151+154
- Recombinant frequency =  $(151+154)/2839$
- $305/2839 = 10.7\%$  recombinant frequency
- Linked in cis conformation
  - $pr^+ vg^+ / pr\ vg$

## Null hypothesis: the genes are unlinked

- $H_0$ : based on Mendel's first law of equal segregation the observed progeny fits a 1:1:1:1 ratio of phenotypes

$pr^+ \cdot vg^+ : pr^- \cdot vg^+ : pr^+ \cdot vg^- : pr^- \cdot vg^-$

| Class                               | O    | E      | (O-E) <sup>2</sup>     | (O-E) <sup>2</sup> /E |
|-------------------------------------|------|--------|------------------------|-----------------------|
| $pr^+ \cdot vg^+$                   | 1339 | 709.75 | (629.25) <sup>2</sup>  | 557.88                |
| $pr^- \cdot vg^+$                   | 1195 | 709.75 | (485.25) <sup>2</sup>  | 331.76                |
| $pr^+ \cdot vg^-$                   | 151  | 709.75 | (-558.75) <sup>2</sup> | 439.87                |
| $pr^- \cdot vg^-$                   | 154  | 709.75 | (-555.75) <sup>2</sup> | 435.16                |
| Total                               | 2839 |        | Chisquare=             | 1764.82               |
| For 4 phenotypic classes df= 4-1= 3 |      |        |                        |                       |

# Linked?

- So  $df = 3$ , chi-square = 1764.82
- Rejection level  $p=0.05$  the  $X^2 = 7.815$ . In fact on the chi-square table for  $p=0.005$  the  $X^2 = 12.838$
- **Conclusion:** given the chi-square = 1764.82 and  $df=3$  a deviation from the expected 1:1:1:1 ratio at least this large would occur by chance alone much less than 5% of the time so we reject the null hypothesis that the observed ratio of  $pr^+ \cdot vg^+ : pr^- \cdot vg^+ : pr^+ \cdot vg^- : pr^- \cdot vg^-$  is 1:1:1:1.



# Another Example: from Mendel

| phenotype       | observed | expected proportion | expected number |
|-----------------|----------|---------------------|-----------------|
| round yellow    | 315      | 9/16                | 312.75          |
| round green     | 101      | 3/16                | 104.25          |
| wrinkled yellow | 108      | 3/16                | 104.25          |
| wrinkled green  | 32       | 1/16                | 34.75           |
| total           | 556      | 1                   | 556             |

# Finding the Expected Numbers

- You are given the observed numbers, and you determine the expected proportions from a Punnett square.
- To get the expected numbers of offspring, first add up the observed offspring to get the total number of offspring. In this case,  $315 + 101 + 108 + 32 = 556$ .
- Then multiply total offspring by the expected proportion:
  - expected round yellow =  $9/16 * 556 = 312.75$
  - expected round green =  $3/16 * 556 = 104.25$
  - expected wrinkled yellow =  $3/16 * 556 = 104.25$
  - expected wrinkled green =  $1/16 * 556 = 34.75$
- Note that these add up to 556, the observed total offspring.

# Calculating the Chi-Square Value

- Use the formula.
- $X^2 = (315 - 312.75)^2 / 312.75$   
+  $(101 - 104.25)^2 / 104.25$   
+  $(108 - 104.25)^2 / 104.25$   
+  $(32 - 34.75)^2 / 34.75$

$$= 0.016 + 0.101 + 0.135 + 0.218$$

$$= 0.470.$$

$$X^2 = \sum \frac{(obs - exp)^2}{exp}$$

# D.F. and Critical Value

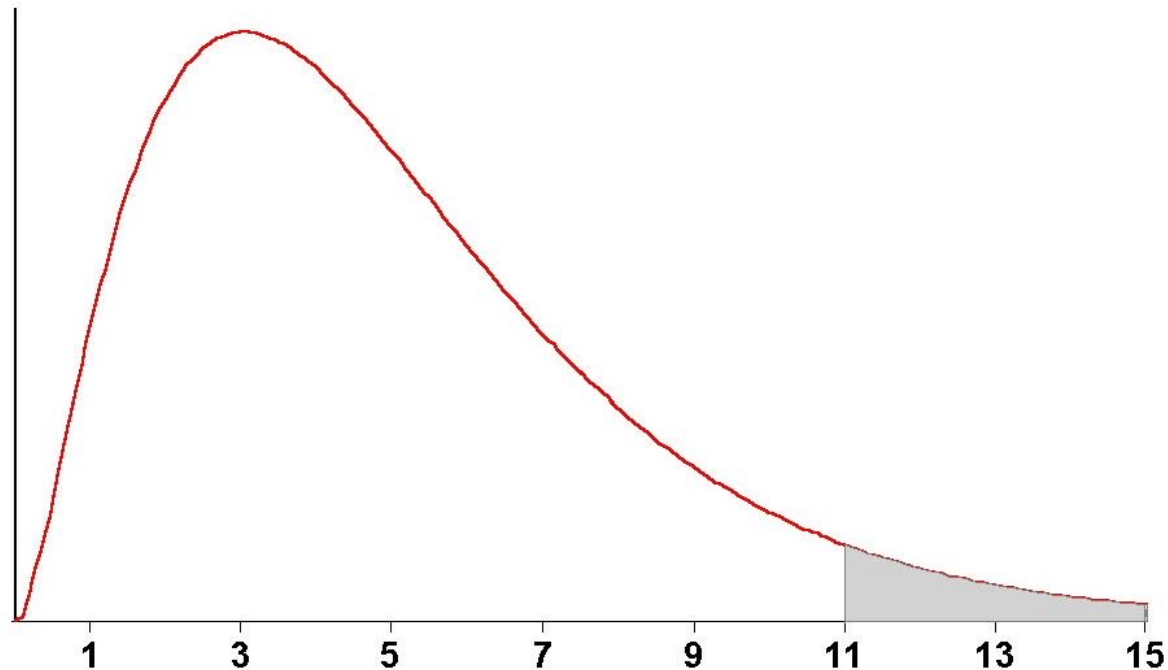
- Degrees of freedom is 1 less than the number of classes of offspring. Here,  $4 - 1 = 3$  d.f.
- For 3 d.f. and  $p = 0.05$ , the critical chi-square value is 7.815.
- Since the observed chi-square (0.470) is less than the critical value, we fail to reject the null hypothesis. We accept Mendel's conclusion that the observed results for a  $9/16 : 3/16 : 3/16 : 1/16$  ratio.
- It should be mentioned that all of Mendel's numbers are unreasonably accurate.

**Table 5-2**  
**Critical Values of the  $\chi^2$  Distribution**

| df \ $p$ | 0.995 | 0.975 | 0.9   | 0.5    | 0.1    | 0.05   | 0.025  | 0.01   | 0.005  | df |
|----------|-------|-------|-------|--------|--------|--------|--------|--------|--------|----|
| 1        | .000  | .000  | 0.016 | 0.455  | 2.706  | 3.841  | 5.024  | 6.635  | 7.879  | 1  |
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| 9        | 1.735 | 2.700 | 4.168 | 8.343  | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 9  |
| 10       | 2.156 | 3.247 | 4.865 | 9.342  | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 10 |
| 11       | 2.603 | 3.816 | 5.578 | 10.341 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 11 |
| 12       | 3.074 | 4.404 | 6.304 | 11.340 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 12 |
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| 15       | 4.601 | 6.262 | 8.547 | 14.339 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 15 |

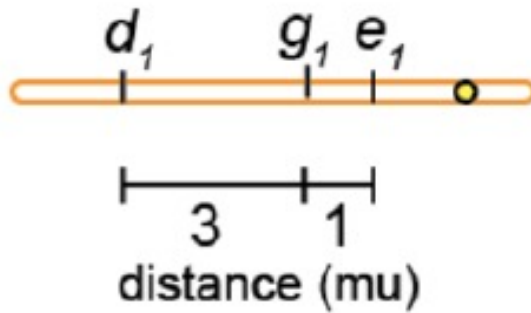
# Chi-Square Distribution, p. 2

- You see that there is a range here: if the results were perfect you get a chi-square value of 0 (because  $\text{obs} = \text{exp}$ ). This rarely happens: most experiments give a small chi-square value (the hump in the graph).
- Note that all the values are greater than 0: that's because we squared the  $(\text{obs} - \text{exp})$  term: squaring always gives a non-negative number.
- Sometimes you get really wild results, with obs very different from exp: the long tail on the graph. Really odd things occasionally do happen by chance alone (for instance, you might win the lottery).

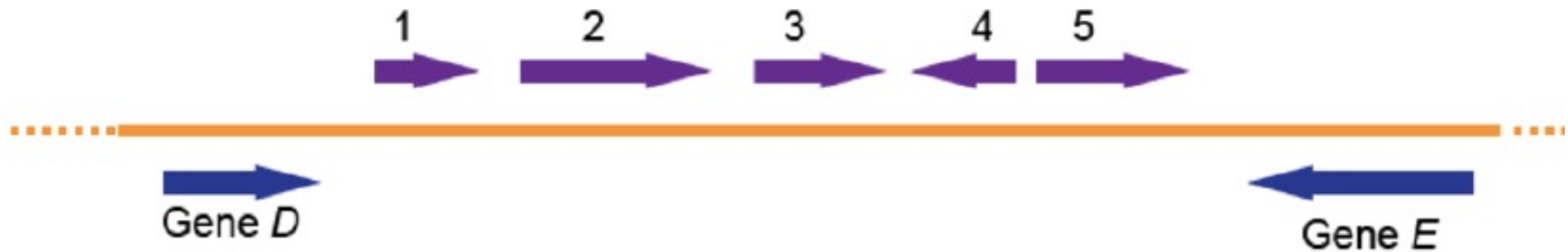


# Which is the right gene??

**A**



**B**

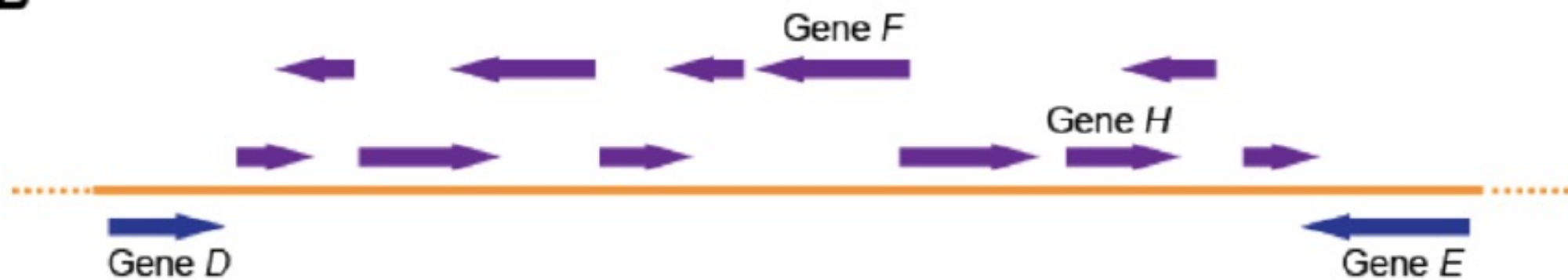


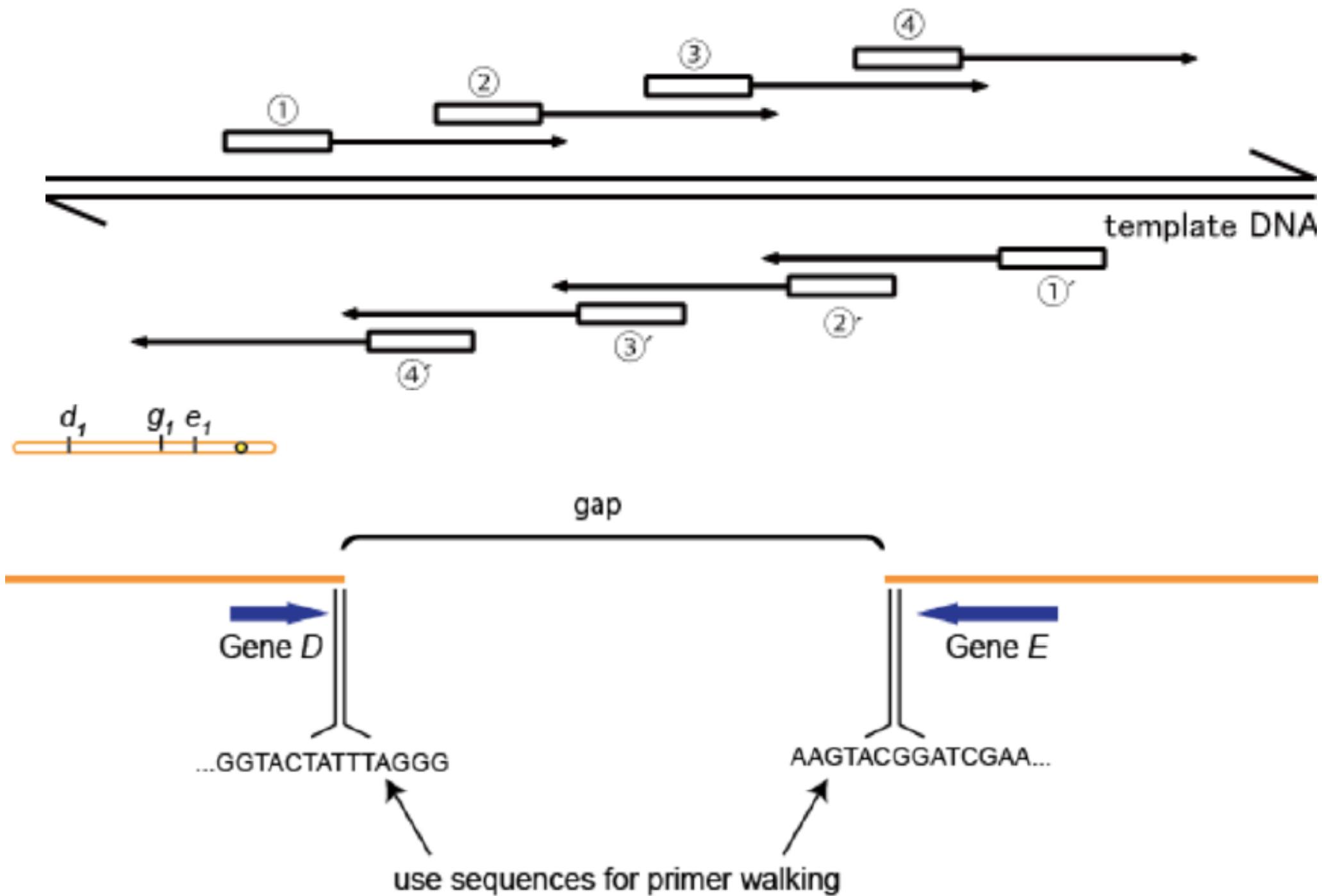


**A**



**B**





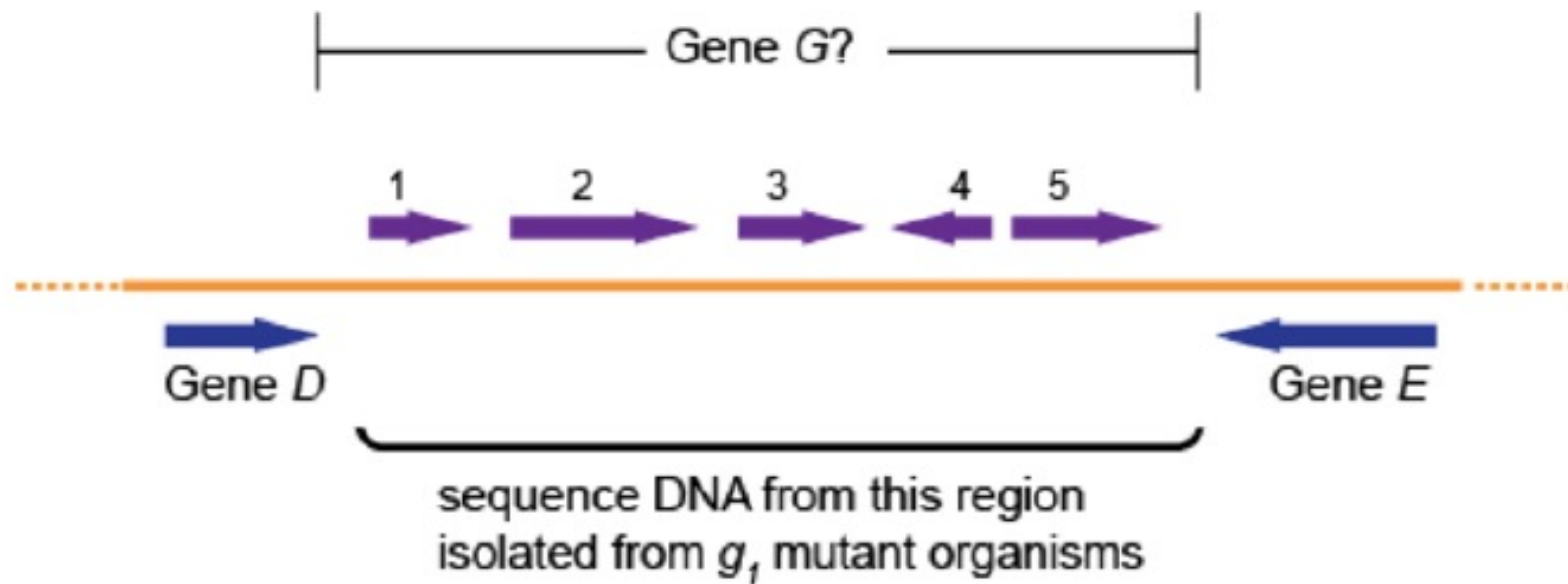
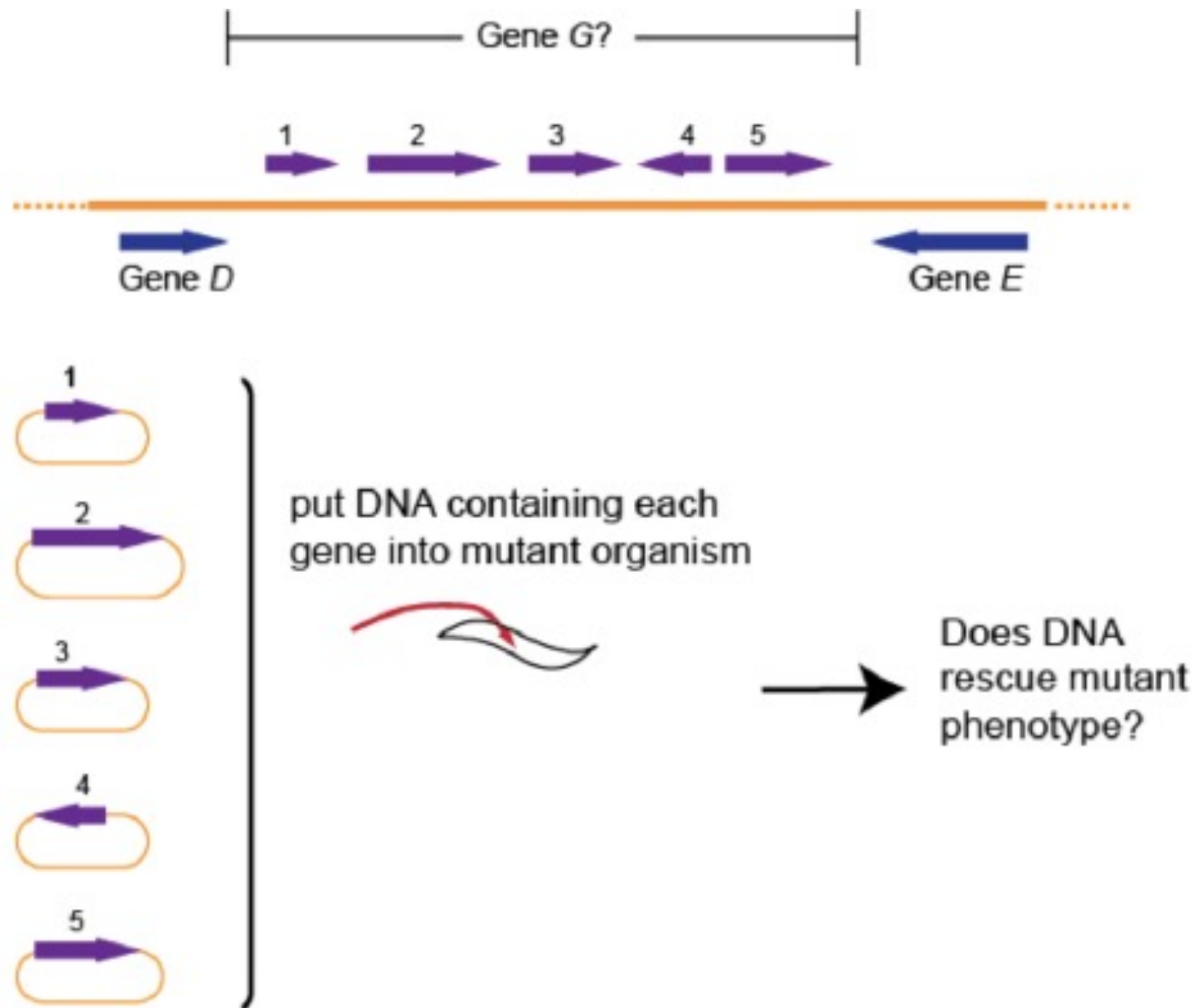


Diagram illustrating the DNA sequence of the  $g_1$  mutant region, focusing on exon 5.

wild type sequence: . . . GAT TCT ATC GTT AAT GTT GTT GAA GAC GAT GTT AAG TAC . . .

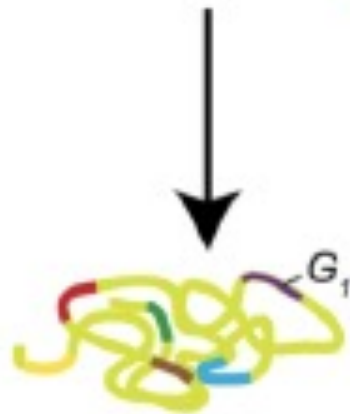
$g_1$  sequence: . . . GAT TCT ATC GTT AAT GTT GTT GAA GAC GAT GTT TAG TAC . . .

A red arrow points to the TAG codon in the  $g_1$  sequence, indicating a premature stop codon mutation.

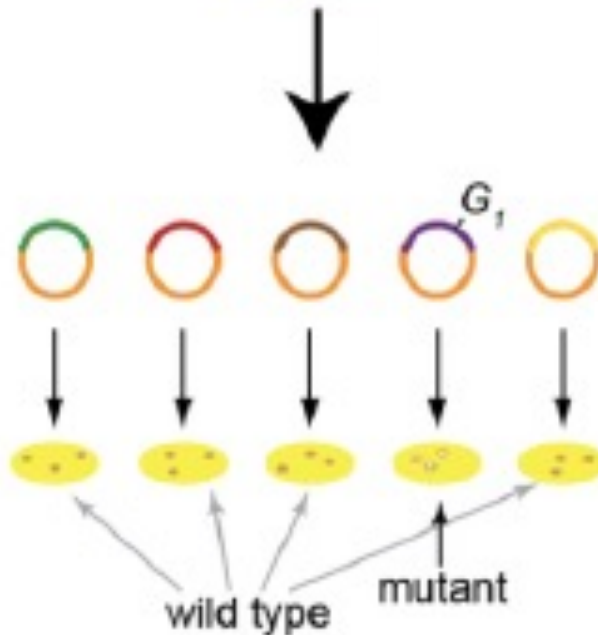




**dominant clones**



**Cut genomic DNA**

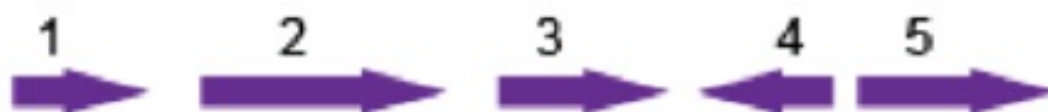


**Create library of genomic DNA  
Insertions in plasmid vector**

**Transform into wild type yeast**

**Which plasmid results in a mutant  
phenotype**

Gene *G*?



Gene *D*

Gene *E*

purify proteins



assay biochemical activity

“Today, if you’re a criminal and you’ve left your DNA at the crime scene, you may as well turn yourself in now. We will catch you,” Mark Lindquist said at a June 22 news conference. The Pierce County, Wash., prosecutor was announcing the arrest of Gary Hartman for the 1986 killing of 12-year-old Michella Welch.

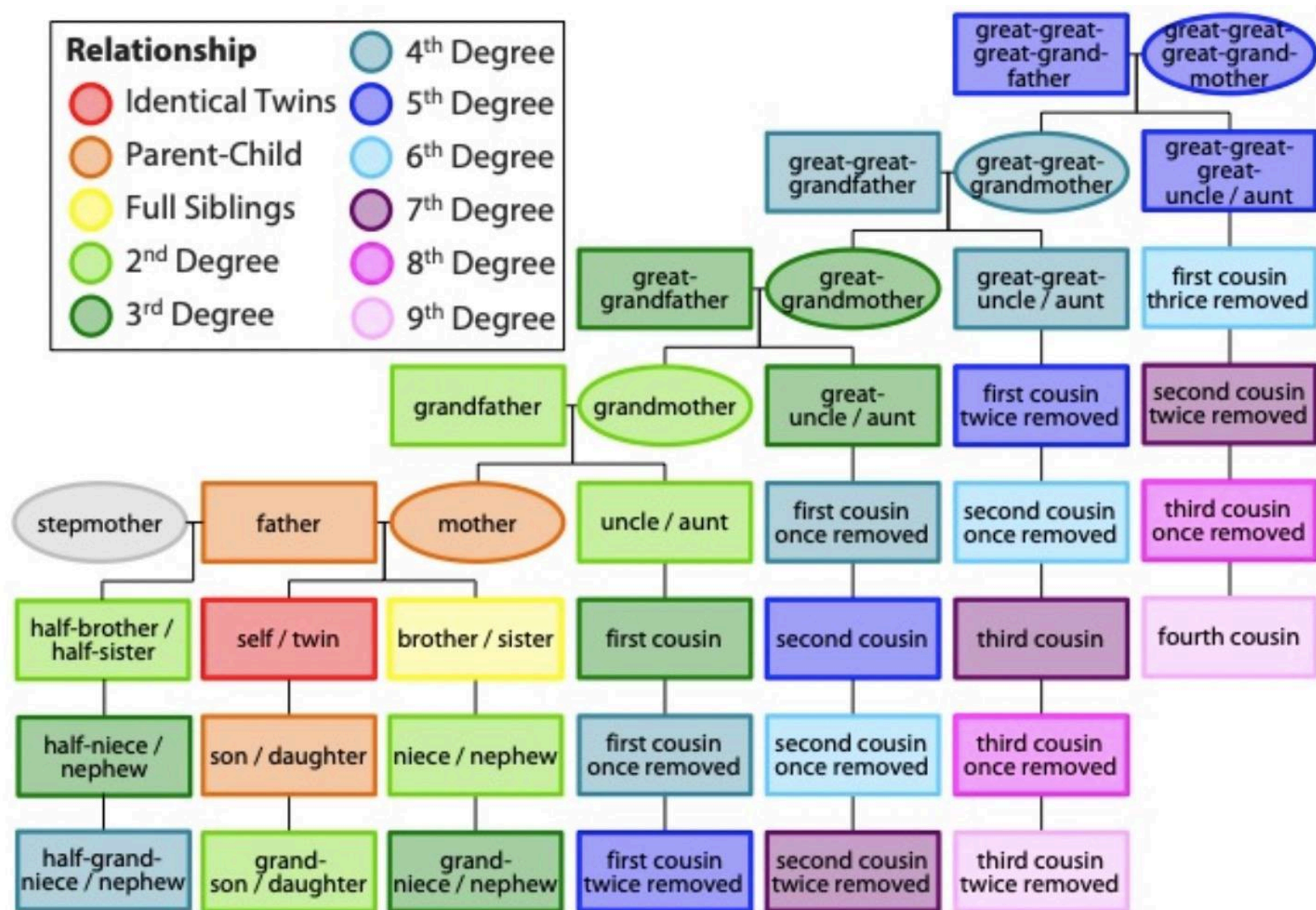
## Crime solvers embraced genetic genealogy

The Golden State killer case was just the beginning



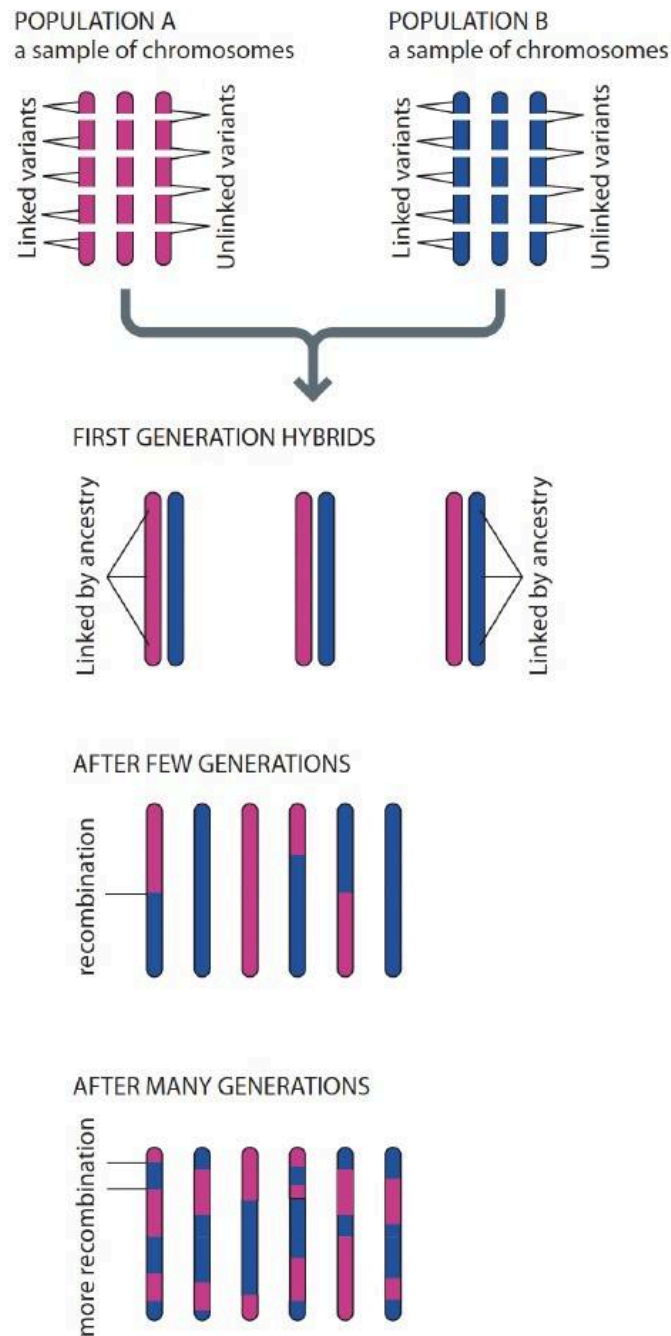
**CAUGHT BY DNA** Sacramento County Sheriff Scott Jones (left) discusses the April arrest of Joseph James DeAngelo





**Figure 1:** Pedigree showing the degrees of relatedness, as defined by the expected amount of shared DNA. Each relationship is defined with respect to the red “self / twin” box.

# Chromosomes are mosaics of ancestral chromosomes



Ancestor

Present-day

